Quantitative Kriging Neighbourhood Analysis for the Mining Geologist — A Description of the Method With Worked Case Examples

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ABSTRACT

Ordinary kriging and non-linear geostatistical estimators are now well accepted methods in mining grade control and mine resource estimation. Kriging is also a necessary step in the most commonly used methods of conditional simulation used in the mining industry. In both kriging and conditional simulation, the search volume or 'kriging neighbourhood' is defined by the user. The definition of this search can have a very significant impact on the outcome of the kriging estimate or the quality of the conditioning of a simulation. In particular, a neighbourhood that is too restrictive can result in serious conditional biases. The methodology for quantitatively assessing the suitability of a kriging neighbourhood involves some simple tests (which we call 'Quantified Kriging Neighbourhood Analysis' or QKNA) that are well established in the geostatistical literature. The authors argue that QKNA is a mandatory step in setting up any kriging estimate, including one used for conditioning a simulation. Kriging is commonly described as a 'minimum variance estimator' but this is only true when the neighbourhood is properly defined. Arbitrary decisions about searches are highly risky, because the kriging weights are directly related to the variogram model, data geometry and block/sample support involved in the kriging. The criteria to look at when evaluating a particular kriging neighbourhood are the following:

1. the slope of the regression of the 'true' block grade on the 'estimated' block grade;
2. the weight of the mean for a simple kriging;
3. the distribution of kriging weights themselves (including the proportion of negative weights); and
4. the kriging variance.

Outside of the technical geostatistical literature, there is little in the published domain to describe the nature of QKNA and no practical presentation of case examples. In this paper we attempt to redress this by setting out the calculations required for QKNA and defining some approaches to interpreting the results. Several practical worked mining case examples are also given. Finally some comments are made on using the results of QKNA to assist with block size selection, choice of discretisation and mineral resource classification decisions.

INTRODUCTION

This paper presents the methodology for quantitatively assessing the suitability of a kriging neighbourhood: ie the combination of the search strategy and block definition used in a kriging. In this paper 'kriging' refers to ordinary kriging (OK), unless otherwise indicated and the process of assessing a kriging neighbourhood (for any kind of kriging) is referred to as 'Quantified Kriging Neighbourhood Analysis' or QKNA. The authors argue that QKNA is a mandatory step in setting up any kriging estimate, including one used for conditioning a simulation.

The criteria for assessing the quality of kriging given a specified kriging neighbourhood (or 'neighbourhood') are well established. However, outside of the specialist geostatistical literature (Armstrong, 1998; David, 1977; Rivoirard, 1987; Chiles and Delfiner, 1999), there is little in the published domain to describe QKNA or to guide geologists in implementation. In this paper an attempt is made to redress this by setting out the calculations required for QKNA and defining some approaches to interpreting the results. Several practical worked mining case examples are also given. Finally, some comments are made on using the results of QKNA to assist with block size selection, choice of discretisation and mineral resource classification decisions.

This paper assumes the reader has a basic understanding of linear geostatistics. Armstrong (1998), Chiles and Delfiner (1999), Isaaks and Srivastava (1989) or Journel and Huijbregts (1978) can be referred to for the required background on variograms and kriging.

MOTIVATION

The motivation for QKNA

Ordinary kriging (OK) and non-linear geostatistical estimators, including uniform conditioning and multiple indicator kriging, are now widespread and routine methods in mine resource estimation and grade control. In this paper 'kriging' refers to OK, unless otherwise indicated. Kriging (Matheron, 1962, 1963a, 1963b; Journel and Huijbregts, 1978) is also a necessary step in the main methods of conditional simulation used in the mining industry, eg sequential Gaussian simulation (SGS), turning bands (TB) and sequential indicator simulation (SIS). Conditional simulation (Journel, 1974; Lantuejoul, 2002) is now being utilised by mining geologists in grade control, resource estimation and risk analysis applications.

In both kriging and conditional simulation, the neighbourhood is defined by the user (or at least it should be: in some cases a 'black box approach' may involve accepting default parameters). Arbitrary specification of the neighbourhood is very risky because the kriging weights are directly related to the variogram model, data geometry and block/sample support involved in the kriging. Whilst kriging is commonly and correctly described as a 'minimum variance estimator' this is only true when the neighbourhood is properly defined. This necessitates an objective method to assess what constitutes an 'appropriate' neighbourhood.

Misconceptions about kriging searches

There is a widely held misconception that 'searching to the range of the variogram' is a good strategy for defining the neighbourhood. The choice of neighbourhood should be influenced more by the slope of the variogram model at short lags and the relative nugget effect (ie the ratio of the nugget variance to the total variance, expressed as a percentage) than by the ranges *per se*.

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In fact, as the range of a variogram approaches zero ('pure nugget effect') it can be shown that the neighbourhood required for good estimation will progressively get larger. In the case of 'pure nugget', correlation between *any* two points in a domain is zero. Therefore, samples located within any limited search neighbourhood will be uncorrelated to the true grade of the block! In other words, local estimation is risky and will be increasingly riskier as we define progressively smaller neighbourhoods. In the case of 'pure nugget', the most reliable estimate will be made with the largest number of samples. In fact in this case searching the whole domain will be the 'minimum estimation variance solution'.

On the other hand, when the relative nugget effect approximates zero and the range is very long relative to the block dimensions, the closest samples are highly correlated to the true block grade. Therefore, only nearby samples will be required to ensure that a good estimate is made when kriging and relatively restricted searching can produce a 'minimum estimation variance solution'.

When no QKNA is performed, choice of search is sometimes made on the basis of comparing the output results from several estimations with different searches. In the authors' experience the danger in this approach is that the most financially attractive result will often be selected. Since the most financially attractive estimate is generally a product of the most restricted search, the risk is that the most conditionally biased result tends to be selected.

Consequences of overly-restricted neighbourhoods: conditional bias

The neighbourhood we choose has a very significant impact on the outcome of the kriging estimate. In particular, a neighbourhood that is too restrictive will result in serious conditional biases (Krige, 1994, 1996a, 1996b). Understanding that conditional biases can be eliminated by a regression approach was the primary original contribution of D G Krige to resource estimation (Krige, 1951), and marked the beginning of modern approaches to resource estimation, leading to geostatistics. The interpolation method called kriging is simply a linear regression solution to the grade interpolation problem.

The awkward phrase 'conditional unbiasedness' has been used historically by geostatisticians to describe the following property of kriging: blocks estimated to have a certain grade Z^*_V will, on average, have that grade (David, 1977).

Figure 1 shows the well-known regression between estimated block grades Z^*_V and true block grades Z_V (see Journel and Huijbregts, 1978 for more detailed discussion). The correlation between true and estimated grades is always imperfect in any practical situation, ie any non-exhaustive drill sampling, and the regression line will usually be flatter than $Y = X$ ($Z_V = Z^*_V$). This implies that using any 'data honouring estimator', like nearest neighbour (or for that matter inverse distance estimators, which as the power increases, approximate nearest neighbour), will result in conditional biases, specifically:

1. estimated grades Z^*_V greater than the mean grade will, on average, be too high; and
2. estimated grades Z^*_V less than the mean grade will, on average, be too low.

The lack of 'perfect' or 'exhaustive' information implies that the correlation between estimates and true block grades will be imperfect. This in turn implies overstatement of high grade and understatement of low grades by local estimators, on average. As the spatial correlation of grades deteriorates (ie high relative nugget and/or presence of a short range structure) the slope of the regression gets progressively less than one. Because it is important (for example in any material classification) that an estimator results in the correct average grade being estimated for blocks in various grade classes, smoothing is required to ensure that the regression slope is as close as practicable to one. Note: the four quadrants marked on Figure 1 denote the correct and incorrect classification of material for a given cut-off grade.

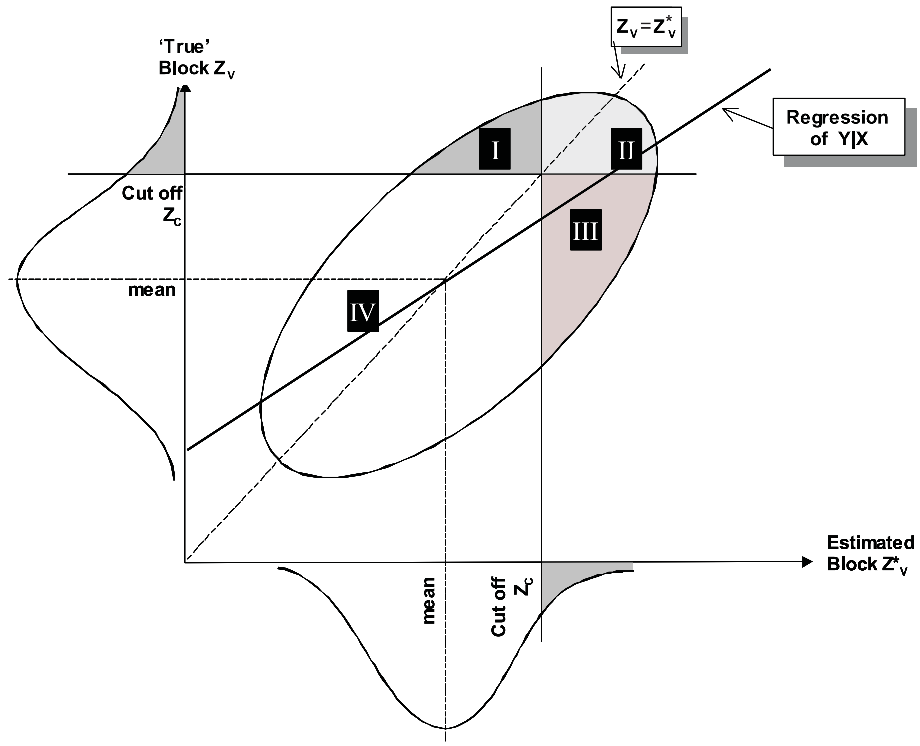

FIG 1 - Information effect diagram showing linear regression. Quadrants II and IV are correct classification of waste and ore, respectively; while quadrants I and II are corresponding incorrect classifications.

In looking at Figure 1, it is obvious that in practice we never know the 'true' grades of blocks. However, we can infer the relationship between true block values and estimated block values under certain assumptions. A critical assumption is that the variogram is modelled reliably and represents the domain of interest adequately, ie the assumption of intrinsic stationarity (see Journel and Huijbregts, 1978, Chapter II) is reasonable. The assumption of intrinsic stationarity enables the calculation of covariances, thus correlations, between any specified supports, for example between true and estimated blocks. We must also assume that a linear regression can capture the essence of the relationship between true and estimated block grades.

Under the assumptions stated previously (the variogram is valid and the regression is linear), it is possible to calculate the main parameters of the regression between estimated and true block grades, given a certain variogram model, informing data set, block size and neighbourhood. Note that the actual scatter plot (as illustrated and summarised by the ellipse in Figure 1) cannot be plotted because we don't know individual true block grades! What can be known, however, are the slope of the regression and the covariance (thus correlation coefficient) between estimated and true block grades.

The process of QKNA is, in essence, one of adjusting the neighbourhood to arrive at good regression statistics in order to reduce or eliminate conditional bias. It is important to appreciate that this will *necessitate* smoothing. This necessity to smooth is a consequence of the 'information effect' (Journel and Huijbregts, 1978) which implies that the estimated block values will have a lower variance than the true block values. The only way to obtain block estimates that are equivalent to the real ('true') block grades is to base the estimate on exhaustive data: ie 'mine the orebody out with a drill rig'. In all realistic cases, some smoothing is necessary and the task is to minimise conditional bias. In other words: smoothing is the price we must pay for non-exhaustive information.

The question is: 'how much do we smooth to ensure conditional unbiasedness?' The answer to this question requires analysis (QKNA), not guesswork.

EVALUATION CRITERIA

The criteria to look at when evaluating a kriging

The objective of QKNA is to determine the combination of search neighbourhood and block size that will result in conditional unbiasedness. The criteria to consider when evaluating a particular kriging neighbourhood, in the order of priority used in practice by the authors, are:

1. the slope of the regression of the 'true' block grade on the 'estimated' block grade;
2. the weight of the mean for a simple kriging;
3. the distribution of kriging weights themselves (including the proportion of negative weights); and
4. the kriging variance.

Calculation of required parameters

Any useful kriging program will provide kriging variance for each estimated block. The most important criterion, the slope of the regression, can usually be obtained from kriging programs. It may be necessary to use a 'debug' option, where blocks are kriged individually and the weights and various other statistics are reported to an output file. In some commercial mining software, the output requires further post-processing to obtain the slope of regression.

If the slope of the regression cannot be obtained in your software, we recommend that you liaise with the developers to have this simple enhancement made available.

The weight of the mean in a simple kriging is not so easy to obtain because it is not offered as an option in some mine planning software.

Slope of the regression

Under the assumptions stated previously (that the variogram is valid and the regression is linear), it is possible to calculate the main parameters of the regression between estimated and true block grades. We repeat that the actual scatter plot (as illustrated and summarised by the ellipse in Figure 1) cannot be plotted because we don't know individual true block grades! What can be calculated is the covariance (thus correlation coefficient) between estimated and true block grades. The slope is given in terms of this covariance and the variance of the estimated blocks by the expression:

$$a = \frac{Cov(Z_v, Z_v^*)}{Var(Z_v^*)}$$

where:

a is the slope of the regression

Z_v is the true block grade

Z_v^* is the estimated block grade

Note that the value of a is often directly given by kriging programs. However, we include in a technical appendix more details on the derivation of a for interested readers.

Ideally, the slope of the regression a should be very close to 1.0 and thus imply conditional unbiasedness. In these circumstances, the true grade of a set of blocks should be approximately equal to the grade predicted by the kriged estimation. The slope and it's interpretation are discussed more fully by Krige (1994; 1996a) and Rivoirard (1987).

The slope should be 1.0 for conditional unbiasedness. A rewriting of the expression for the slope in terms of correlation coefficient ρ is possible:

$$a = \rho \frac{\sigma_{Z_V}}{\sigma_{Z_V^*}}$$

where:

a is the slope of the linear regression

ρ is the linear (Pearson) correlation coefficient

σ_{Z_V} is the standard deviation of true block grades

$\sigma_{Z_V^*}$ is the standard deviation of estimated block grades

From the above expression we can see that even for slope equalling one, the correlation may be less than one (because the smoothing effect of kriging necessitates that the variability of estimates is lower than that of true blocks). We discuss the interpretation of slope further in the section below on resource classification, but note here that no statistical criteria should ever be the sole basis of resource categorisation.

Weight of the mean for simple kriging

Instead of performing an ordinary kriging, where the sum of the weights is set to one, we can run simple kriging (SK) where the sum of the weights is not constrained to add up to one. The remaining weight is allocated to the mean grade of the domain ('the weight of the mean') and is an inversely proportional index of 'screen effect'. A sample is said to be 'screened' if another sample lies between it and the point being estimated, in which case the estimation weight of the screened sample is reduced. If the variogram indicates high continuity, screen effect will be pronounced; conversely, high nugget effect (or significant short scale structure) implies that weights will be spread far away from a block to reduce conditional bias and minimise estimation variance.

SK is also called 'kriging with known mean', and is based on the assumption that the global mean grade is known and equal to *m*:

$$Z_V^{*SK} - m = \sum \lambda_i^{SK}(Z(x_i) - m)$$

where:

$\lambda_m = 1 - \sum \lambda_i^{SK}$ is the weight assigned to the global mean grade or 'weight of the mean'

The weight of the mean for a given neighbourhood, denoted λ_m, gives a clear idea of the quality of kriging because it is a measure of the weakness of the screen effect. The larger λ_m is, the weaker we expect the 'screen effect' to be. Consequently, all things being equal, it is better to choose a larger kriging neighbourhood as λ_m increases, (Rivoirard, 1987). As a general rule, we prefer the weight of the mean to be close to zero. The objective in QKNA is to obtain the combination of the best slope with a minimised weight of the mean.

Note that the use of SK here is solely for QKNA and that, in general, the stationarity assumptions of SK are not suited to mining grade estimation. It can be shown that OK is exactly the same as SK when *m* is replaced by its kriged estimation (Armstrong, 1998).

Kriging weights (and negative weights)

If we can expand the neighbourhood and still assign a meaningful positive weight to the incremental samples thus obtained, then the search is too restrictive. At the margins of an optimised search, kriging weights should be very small ('trivial'), or even slightly negative.

Most mining grade variables are certainly not spatially smooth, ie there is at least some nugget effect. In these circumstances, a 'screen effect' can be expected and at some distance negative weights will be observed. The distance we need to search before negative weights are encountered progressively increases as the effective nugget effect increases. In the case of 'pure nugget' every sample found gets equal weight (1/N) no matter how far we search. Negative weights are not problematic if they represent a small proportion (say <5 per cent) of total weight.

The authors advise against modified kriging algorithms that adjust negative weights (eg Deutsch, 1996) or set them to zero, since such approaches will assure conditional bias.

Kriging variance

The minimum variance solution to the ordinary kriging equations results in an estimation variance, also called the kriging variance (KV). Refer to the appendix for the expression for KV.

A mapping of KV gives an idea of the relative quality of estimation (mainly in terms of data density and geometry), but maps of regression slope may be more useful.

Determining minimum and maximum numbers of data

The minimum number of data used in an estimate ('Nmin') can be studied as a variable in a QKNA exercise. However, as a 'rule of thumb' using less than ten - 12 samples is not recommended, especially in the presence of any appreciable short-scale structure or nugget effect. The default value for Nmin in software systems may be as low as 1 or 2: this is effectively a nearest a neighbour interpolation and in most cases this cannot be technically defended.

There are cases where the search will be set larger than strictly necessary to ensure that an adequate number of data are used in less sampled areas of a domain (eg at the edges). In these cases, ability to specify a maximum number of data to used in an estimate ('Nmax') allows the search to be relaxed 'automatically'.

Another practical implementation question of interest is the use of quadrant or octant searches. The kriging estimate does perform a degree of declustering, but such strategies are often still very important.

APPLICATIONS

Application of QKNA to kriging estimation

The optimal solution would be to 'tune' the neighbourhood for every block estimated but this is obviously not practical. Therefore the objective is to find best compromise solution: a neighbourhood is defined for a domain that will be adequate for as many blocks as possible. If differences between optimal neighbourhoods and this 'compromise neighbourhood' are too big (as can happen when data density is variable across a domain) then there may be an argument to further divide estimation area and perform multiple runs.

The kriging weights depend upon the data in the sense that the variogram model we choose is intimately linked to the histogram and spatial continuity of the samples themselves; however, the kriging equations contain no direct reference to the data values themselves. This means the set of weights obtained for a given sampling/block geometry and a specified variogram model will always be the same, regardless of the sample grades. Because of this property, we need only test a range of 'typical' data configurations in order to determine the optimal search (we don't have to test every block). The authors recommend testing a range of blocks as follows:

1. well informed blocks (amidst plenty of sampling information, especially with samples internal to the block);
2. less well informed blocks but still with data surrounding the block; and
3. poorly informed blocks, including:
   - blocks with no internal samples; and
   - blocks with no samples in certain directions (ie blocks at the edges of domains).

Block size determination

It is important to understand that the block size is critical in all cases where a cut-off will be applied to an estimate (ie unless the estimate is to be used only in a global sense and without a cut-off). There is a long bibliography of warnings against estimation of small blocks (for example Armstrong and Champigny, 1989; Ravenscroft and Armstrong, 1990; Royle, 1979; Vann and Guibal, 2000). The question is, how small is too small?

By running QKNA for a range of block sizes, in relatively well- and poorly-informed instances, quantitative determination of appropriate block sizes is possible. The results of such an analysis will usually show that the slope of the regression and weight of the mean rapidly become unacceptable as the block size reduces, except for those blocks actually containing samples. As a general summary, the block size needs to increase as the nugget (and other short-scale discontinuities) increases. It is unusual for blocks appreciably smaller than half the drilling grid dimensions to yield acceptable QKNA results, unless the grade continuity is *very* high (ie very low nugget and long ranges).

Discretising the block in kriging

The discretisation in block kriging is used to calculate the point-block average values of variogram (or covariance) functions, ie $\bar{\gamma}(x_i, V)$ or $\overline{C}(x_i, V)$. The general process for determining discretisation of a block involves iterative calculation of $\bar{\gamma}(V,V)$ or equivalently $\bar{C}(V,V)$ with a range of discretisations. Ideally, we calculate for each discretisation several times, moving the origin of the discretisation grid each time. Stable results indicate that the discretisation is adequate. The number of discretised points should be compatible with the dimension of the block in units of composite-length in the direction approximately parallel to the drilling, ie when a composite can no longer be reasonably considered as 'a point in space' but rather a regularised variable (its dimension is no longer '0' at the scale of the block). For example, in an open pit situation with steeply inclined drilling, a block 10 m high would be discretised to 2 in the Z (vertical) when using 5 m composites, but to 5 in the vertical when using 2 m composites or 1 in the vertical for 10 m composites.

In general, higher discretisations are better, with the only disadvantage being some computing speed consequences. Note that the speed penalty for higher discretisation when kriging is *not* as severe as it is in IDW interpolation, where discretisation to *n* points requires *n* estimates. In kriging the discretisation is used solely for the calculation of $\bar{C}(V,V)$ and $\bar{C}(x_i,V)$.

Table 1 shows results of a sensitivity analysis of discretisation on $\bar{C}(V,V)$ (for the same variogram and block size used in case Study A, later). Figure 2 shows a graph of the impact of discretisation on the resulting range of $\bar{C}(V,V)$ values.

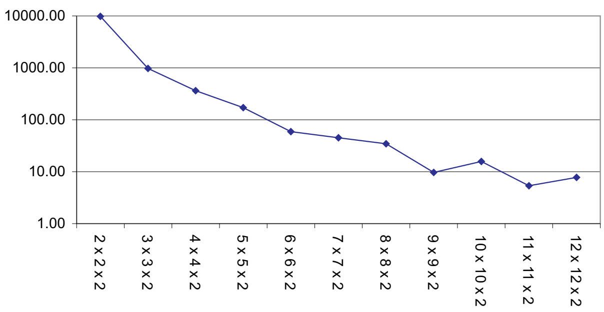

FIG 2 - Visualisation of example of sensitivity of $\bar{C}(V,V)$ to discretisation. Note that the variance for results, plotted on the Y-axis does not show significant continuing downward trend beyond about 6 × 6 × 2. In this case 6 × 6 × 2 or 7 × 7 × 2 would be acceptable.

Application of QKNA to conditional simulation

Conditional simulation is widely applied by mining geologists for grade control, resource estimation and risk analysis. Kriging is also a necessary step in the methods of conditional simulation most commonly used in the mining industry: sequential Gaussian simulation (SGS) and turning bands (TB) and sequential indicator simulation (SIS). Refer to Chiles and Delfiner (1999), Goovaerts (1997) and Lantuejoul (2002) for details on simulation algorithms.

In sequential algorithms, such as SGS and SIS, the kriging step is at the core of the algorithm and conditioning of the simulation is by construction. In TB a non-conditional simulation is performed, followed by an explicit conditioning step by kriging. In both cases, the quality of the kriging is determined substantially by the adequacy of the neighbourhood used. The authors note that in more 'automated' approaches to simulation sometimes employed in grade control the definition of the neighbourhood may not be transparent to the user.

Application of QKNA to multivariate situations

QKNA can be applied to the multivariate situation as well. It requires a consistent model of co-regionalisation to allow for the modelling of spatial correlation intra-variables (say gold with gold, copper with copper) and inter-variables (say gold with copper). The assumption of stationarity inherent in the use of such a model is more restrictive because it needs to apply to domains that ought to display an acceptable level of statistical homogeneity for *all* variables and their spatial correlations. Depending on the nature of the spatial relationship between the variables under consideration, different models of correlation may be used (intrinsic correlation, linear model of co-regionalisation, complex model of co-regionalisation). For more details on multivariate applications, refer to Wackernagel (1995) and Wackernagel and Grzebyk (1994, 1995).

TABLE 1

Example of sensitivity of $\bar{C}(V,V)$ to discretisation. NB. same variogram model for Case Study A: for each discretisation, 13 calculations of $\bar{C}(V,V)$ were made with randomly changed grid origin.

Discretisation	2 × 2 × 2	3 × 3 × 2	4 × 4 × 2	5 × 5 × 2	6 × 6 × 2	7 × 7 × 2	8 × 8 × 2	9 × 9 × 2	10 × 10 × 2	11 × 11 × 2	12 × 12 × 2
Cvv test1	1.31	1.26	1.24	1.22	1.23	1.24	1.23	1.23	1.23	1.24	1.23
Cvv test2	1.35	1.23	1.29	1.27	1.24	1.23	1.23	1.23	1.23	1.24	1.23
Cvv test3	1.22	1.26	1.24	1.25	1.24	1.25	1.24	1.23	1.24	1.23	1.24
Cvv test4	1.11	1.25	1.27	1.22	1.22	1.23	1.23	1.23	1.23	1.23	1.23
Cvv test5	1.28	1.26	1.28	1.24	1.25	1.25	1.23	1.23	1.23	1.24	1.24
Cvv test6	1.48	1.24	1.26	1.23	1.25	1.23	1.23	1.23	1.23	1.23	1.24
Cvv test7	1.21	1.30	1.24	1.25	1.23	1.23	1.24	1.24	1.23	1.23	1.24
Cvv test8	1.27	1.20	1.27	1.23	1.23	1.23	1.24	1.23	1.24	1.24	1.24
Cvv test9	1.17	1.25	1.26	1.23	1.23	1.24	1.22	1.23	1.23	1.23	1.23
Cvv test10	1.41	1.31	1.26	1.24	1.25	1.24	1.23	1.23	1.24	1.24	1.23
Cvv test11	1.29	1.28	1.22	1.24	1.25	1.23	1.24	1.23	1.23	1.23	1.23
Cvv test12	1.28	1.30	1.26	1.23	1.24	1.24	1.23	1.24	1.23	1.24	1.23
Cvv test13	1.37	1.25	1.27	1.26	1.23	1.24	1.23	1.23	1.24	1.23	1.24
Variance	9807.10	976.01	363.44	170.93	59.07	44.95	34.52	9.65	15.70	5.35	7.74

Some further comments on resource classification

One of the factors which can impact on resource classification that is often not adequately considered is the *scale* of classification, ie the volume of mineralisation that is being classified (Stephenson and Vann, 2000). It is unwise, and generally unnecessary, to classify a Resource or Reserve estimate at the scale of a block (especially when small blocks have been used relative to the sample spacing). The classification process is usually best approached at the scale of domains or significant subsets of domains (Domain A above a certain RL, for example). However, the various statistics resulting from QKNA are very useful as inputs to resource classification along with geology, appraisal of drill spacing and so on.

Associations of slope and estimation quality statistics with the Joint Ore Reserves Committee Code classification of Mineral Resources into Measured, Indicated and Inferred (JORC, 1999) require expert consideration of Competent Persons knowledgeable of the geological, geostatistical and other specifics of the deposit at hand. In particular, output of QKNA *are complementary to other inputs to classification (data quality, geology confidence, etc)*.

The reader is also referred to the JORC Code (JORC, 1999) and Stephenson and Vann (2000) for more guidance.

CASE STUDIES

We present here results of two short case studies to illustrate the outcome of QKNA. In each case a table is given showing the results of QKNA tests for three situations in a deposit:

1. a 'well informed' block, ie one with information on all sides and samples in (or very near to) the estimated block, in other words, an 'ideal case';
2. a 'reasonably informed' block, ie one with less information than a well informed block but still moderately well informed; and
3. a 'poorly informed' block, ie one with less than adequate information, for example no samples in or close to the block, or a lack of samples on one side, etc.

The definition of these three cases usually requires testing of a range of possibilities, ie a number of 'well informed' situations, etc. Results for 'typical cases' are given in our examples. Note that the case studies could also be presented to show how the slope of regression improves by varying the size of the search from a restricted one to a reasonable one, but space considerations preclude this here.

Case A – gold grade estimation in a gold deposit (3D)

Table 2 shows the variogram model for Case Study A and Figure 3 shows a cross sectional view of a well informed block. Table 3 shows the results of QKNA tests for three situations in an Archaean gold deposit: a well informed, reasonably informed and poorly informed block.

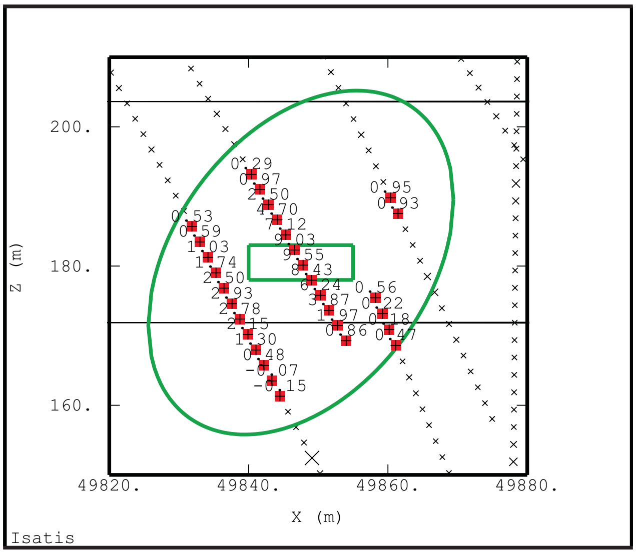

FIG 3 - Cross-sectional view of the estimated block, sample locations, kriging weights and search ellipse selected for the well-informed case in Case Study A. Note, some samples inside the ellipse do not have weights because the maximum number of samples criterion has been met.

TABLE 3

QKNA statistics for Case Study A.

Test result	Well informed	Reasonably informed	Poorly informed
Slope of regression	> 0.93	< 0.8	< 0.6
Correlation coefficient	< 0.9	< 0.7	< 0.5
Weight of the mean	< 15%	> 25%	> 50%
Cumulative sum of negative weights	<0.5%	0%	0%
Kriging standard deviation	~ 0.5	~ 0.7	~ 0.9

The results in Table 3 show that a well informed block has a low weight of the mean (13 per cent) and a slope of regression of 0.93, indicating that the estimation would not be much improved by searching further. Consequently, a trivial amount of negative weight is allocated, and this is not a problem at all! The kriging standard deviation (ie square root of the kriging variance) is significantly lower than for the other two cases. Also, the correlation coefficient indicates that we can expect quite good (but not perfect) reconciliation between estimated and true block grades.

TABLE 2

Variogram model for Case Study A.

	Nugget	Structure 1	Structure 2	Structure 3
		(Spherical)	(Spherical)	(Spherical)
Variance of structure	2.03	1.67	0.07	0.18
Proportion of total sill	51%	42%	2%	5%
Range D1 (m) (Az 330°, Dip 0°)		22	28	16
Range D2 (m) (Az 240°, Dip 60°)		24	70	16
Range D3 (m) (Az 060°, Dip 30°)		200	70	100

The poorly informed block, on the other hand, is only marginally acceptable for Inferred Resources, in the opinion of the authors, because the slope is now close to 0.5 and the weight of the mean is significantly higher than for a 'well informed' block, at 50 per cent of total weight. The correlation expected between estimated and true blocks is now quite poor (less than 0.5). The problem here is that to improve these statistics, we would need to search much further, making the estimate almost 'global' in nature and thus of little local utility. Areas comprised of blocks with statistics worse than these should not really be classified as Resources, in the JORC sense, if any degree of selective mining is envisaged.

Case B – ash content estimation for a coal deposit (2D)

Estimation in two dimensions is appropriate for deposits having a layer or vein-like geometry, eg narrow veins and coal seams (see Bertoli *et al*, 2003, for further details on 2D estimation). This case study is for ash content in a coal deposit. The ash content estimate is used in a decision to send or by-pass a wash plant facility, and is thus the basis of material classification.

Table 4 shows the experimental variogram model for ash accumulation and Figure 4 shows a plan view of a well informed block. The key characteristics of the model fitted to this variogram are: a high nugget effect (~70 per cent) and a spatial structure that is relatively short (500 m) given the data spacing (250 × 250 m).

As a consequence of the poor spatial continuity of ash (evidenced by the variogram model) the weight of the mean tends to be elevated. This motivates searching progressively further away in order to weight a larger number of samples with the objective of reducing conditional bias.

Table 5 shows the results of QKNA tests for ash accumulation (ash content × density × thickness) for three situations in: a well informed, a reasonably informed (ie surrounded by information but no central sample) and a poorly informed block. Each block has dimensions of 125 × 250 m (with the long dimension along strike).

The results in Table 5 were obtained using a search radius of 1500 m down dip and 3500 m along the strike of the coal seam with up to 80 samples for the best informed blocks. The best informed blocks are well estimated (slope of 0.9, correlation on average of 0.70 and up to 0.80) but as soon as the density of information decreases (eg when the block does not contain any samples) the quality of estimation deteriorates rapidly. The conclusion is that the ability to produce reliable local estimates is seriously compromised in such situations. Any attempt to estimate smaller blocks or to use smaller neighbourhoods (in a data honouring strategy) will have costly consequences in terms of material misclassification.

CONCLUSIONS

Kriging is commonly described as a 'minimum variance estimator' but this is only true when the block size and neighbourhood are properly defined. The methodology for quantitatively assessing the suitability of a kriging neighbourhood – QKNA – involves some straightforward tests that are described in detail in this paper. These tests can be implemented in most good mine planning software systems. The definition of the search in both kriging and conditional simulation can have a very significant impact on the outcome of the kriging estimate or the quality of the conditioning of a simulation. In particular, a neighbourhood that is too restrictive can result in serious conditional biases. QKNA at a relatively early stage (wide spaced drilling) can be very important because unrealistic expectations of the grade above an elevated cut-off grade may be raised by estimates based on overly restricted searching.

The authors reiterate that QKNA is, in our view, a mandatory step in setting up any kriging estimate, including one used for conditioning a simulation. In the end, the results of a kriged estimate or conditional simulation can be very sensitive to the neighbourhood and a quantitative approach to selection of the neighbourhood is thus essential.

TABLE 4

Variogram model for Case Study B.

	Nugget	Structure 1 (Spherical)
Variance of structure	25	10
Proportion of total sill	71%	29%
Isotropic range (m)		750

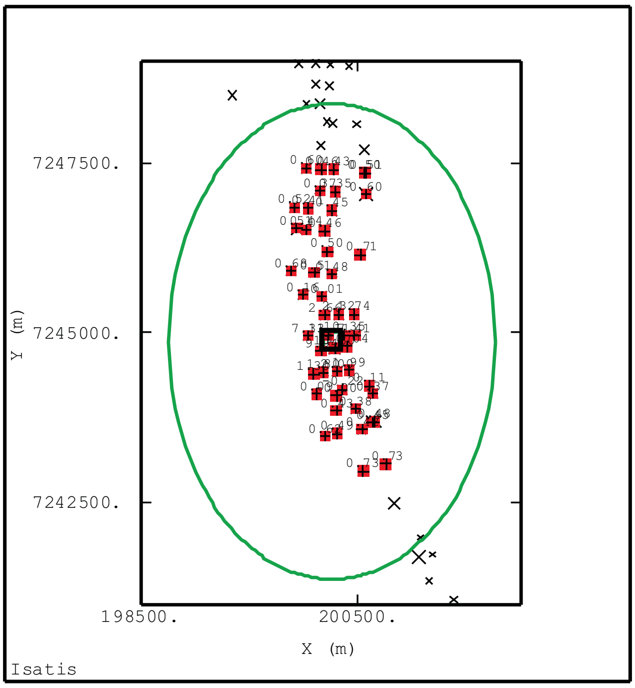

FIG 4 - Plan view of the estimated block, sample locations, kriging weights and search ellipse selected for the well-informed case in Case Study B. Note, some samples inside the ellipse do not have weights because the maximum number of samples criteria has been met.

TABLE 5

QKNA statistics for Case Study B.

Test result	Well informed	Reasonably informed	Poorly informed
Slope of regression	~ 0.90	< 0.70	< 0.30
Correlation coefficient	< 0.70	< 0.50	< 0.20
Weight of the mean	> 30%	> 50%	> 80%
Cumulative sum of negative weights	< 1%	0%	0%
Kriging standard deviation	~ 2.0	~ 2.5	~ 3.0

ACKNOWLEDGEMENTS

The authors would like to acknowledge the improvement made possible by the helpful comments on an early version of this paper by Rick Adams and Ted Coupland of Cube Consulting, Phil Jankowski of Sons of Gwalia, Mike Stewart of MPI Mines and Aaron Tomsett of Quantitative Geoscience. The comments of two anonymous reviewers were appreciated, leading to useful improvement of the arguments presented.

REFERENCES

Armstrong, M, 1998. *Basic Linear Geostatistics*, pp 94 - 95; pp 96 - 97 (Springer: Berlin).

Armstrong, M and Champigny, N, 1989. A study on kriging small blocks, *CIM Bulletin*, 82:128-133.

Bertoli, O, Job, M, Vann, J and Dunham, S, 2003. Two dimensional geostatistical methods: theory, practice and a case study from the 1A nickel deposit, Leinster, Western Australia, in *Proceedings Fifth International Mining Geology Conference,* (The Australasian Institute of Mining and Metallurgy: Melbourne).

David, M, 1977. *Geostatistical Ore Reserve Estimation (Developments in Geomathematics 2),* (Elsevier: Amsterdam).

Deutsch, C V, 1996. Correcting for negative weights in ordinary kriging, *Computers and Geosciences*, 22:765-773.

Chiles, J P and Delfiner, P, 1999. *Geostatistics: Modelling Spatial Uncertainty* (Wiley Inter-Science: New York).

Goovaerts, P, 1997. *Geostatistics for Natural Resources Evaluation*, (Oxford University Press: New York).

Isaaks, E H and Srivastava, R M, 1989. *Applied Geostatistics*, (Oxford University Press: New York).

JORC, 1999. Australasian Code for Reporting of Mineral Resources and Ore Reserves, The Joint Ore Reserves Committee of The Australasian Institute of Mining and Metallurgy, Australian Institute of Geoscientists and Minerals Council of Australia.

Journel, A G, 1974. Geostatistics for conditional simulation of ore bodies, *Economic Geology*, 69:673-687.

Journel, A G and Huijbregts, Ch J, 1978. *Mining Geostatistics*, pp 306 - 307 (Academic Press: London).

Krige, D G, 1951. A statistical approach to some basic mine valuation problems on the Witwatersrand, *J Chem Metall Min Soc S Afr*, 52:119-139.

Krige, D G, 1994. An analysis of some essential basic tenets of geostatistics not always practised in ore valuations, in *Proceedings Regional APCOM: Computer Applications and Operations Research in the Minerals Industries, Slovenia*, pp 15-18.

Krige, D G, 1996a. A basic perspective on the roles of classical statistics, data search routines, conditional biases and information and smoothing effects in ore block valuations, in *Proceedings Conference on Mining Geostatistics,* Kruger National Park, South Africa, pp 1-10 (Geostatistical Association of South Africa).

Krige, D G, 1996b. A practical analysis of the effects of spatial structure and data available and accessed, on conditional biases in ordinary kriging, in *Proceedings Fifth International Geostatistical Congress, Geostatistics Wollongong '96* (Eds: E Y Baafi and N A Schofield) pp 799-810.

Lantuejoul, C, 2002. *Geostatistical Simulation: Models and Algorithms*, (Springer: New York).

Matheron, G, 1962. Traite de geostatistique applique, tome I, *Memoires du Bureau de Recherches Geologiques et Minieres, No. 14,* (Editions Technip: Paris).

Matheron, G, 1963a. Traite de geostatistique applique, tome II: Le krigeage, *Memoires du Bureau de Recherches Geologiques et Minieres, No. 24.* (Editions Technip: Paris).

Matheron, G, 1963b. Principles of geostatistics, *Economic Geology*, 58:1246-1266.

Ravenscroft, P J and Armstrong, M, 1990. Kriging of block models – the dangers re-emphasised, in *Proceedings APCOM XXII,* Berlin, 17 - 21 September 1990, pp 577-587.

Rivoirard, J, 1987. Two key parameters when choosing the kriging neighbourhood, *J Math Geol*, 19:851-856.

Royle, A G, 1979. Estimating small blocks of ore, how to do it with confidence, *World Mining*, April.

Stephenson, P R and Vann, J, 2000. Commonsense and good communication in mineral resource and ore reserve estimation, in *Mineral Resource and Ore Reserve Estimation – The AusIMM Guide to Good Practice*, Monograph 23 (Ed: A C Edwards) pp 13-20 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Vann, J and Guibal, D, 2000. Beyond ordinary kriging – an overview of non-linear estimation, in *Mineral Resource and Ore Reserve Estimation – The AusIMM Guide to Good Practice,* Monograph 23 (Ed: A C Edwards) pp 249-256 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Wackernagel, H, 1995. *Multivariate Geostatistics – An Introduction With Applications* (Springer: Berlin).

Wackernagel, H and Grzebyk, M, 1995. Linear models for spatial or temporal multivariate data, in *Proceedings Sixth International Meeting on Statistical Climatology,* Galway, Ireland, pp 427-429.

Wackernagel, H and Grzebyk, M, 1994. Multivariate analysis and spatial/temporal scales: real and complex models, in *Proceedings XVIIth International Biometric Conference,* Hamilton, Ontario, Canada, Volume 1, pp 19-33.

APPENDIX

The slope of the regression *a is* given in terms of the covariance between estimated and 'true' block grades and the variance of the estimated blocks:

$$a = \frac{Cov(Z_v, Z_v^*)}{Var(Z_v^*)}$$

where:

a is the slope of the regression

Z_V is the true block grade

Z_v^* is the estimated block grade

The variance of the estimates can be calculated from linear geostatistics.

Variance of block estimates

Block grade estimates are written:

$$Z_v^* = \sum_{i=l}^{N} \lambda_i Z(x_i)$$

The variance of estimated block grades can be calculated from the following relationship:

$$Var(Z_V^*) = \sum_{i=1}^{N} \sum_{j=1}^{N} \lambda_i \lambda_j C(x_i, x_j)$$

where:

$C(x_i,x_j)$ is the value of the covariance function between sample location x_i and sample location x_j and the kriging weights associated to these samples are denoted λ_i and λ_j.

Covariance between true and estimated blocks

The covariance between true and estimated blocks $Cov(Z_v, Z_v^*)$ can be calculated (see for example, Armstrong, 1998):

$$Cov(Z_V, Z_V^*) = \sum_{i=1}^{N} \lambda_i \overline{C}(x_i, V)$$

where:

$\overline{C}(x_i, V)$ is the mean value of the covariance function between each sample location x_i and the block to be estimated V.

Note that given the kriging equations which state for any *I*:

$$\sum_{j=1}^{N} \lambda_j Cov(x_i, x_j) - \mu = \overline{C}(x_i, V)$$

we have:

$$Cov(Z_V, Z_V^*) = Var(Z_V^*) - \mu$$

so that**:**

$$a = 1 - \frac{\mu}{Var(Z_v^*)}$$

where:

μ is the Lagrange multiplier used in solving the kriging system written with the covariance function.

Note also that the kriging system and all kriging equations can be expressed in terms of the semi-variogram function ($\gamma(h)$) only. This is particularly interesting for intrinsic random functions for which the covariance is not defined (see Journel and Huijbregts, 1978).

Kriging variance

The expression for the minimum estimation variance (obtained by kriging), also called the kriging variance (KV), is:

$$\sigma_{OK}^2 = \mathrm{Var}(z_V^* - z_V) = \sum_{i=1}^{N} \lambda_i \bar{\gamma}(x_i, V) - \bar{\gamma}(V, V) + \mu$$

All the terms in this expression are defined previously.