

Multivariate iron ore deposit resource estimation – a practitioner’s guide to selecting methods

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When only wide-spaced drilling is available, for example at concept, pre-feasibility and feasibility stages, properly implemented linear estimation (including ordinary kriging) predicts grade–tonnage relationships that are distorted compared to final production estimates (and production). Non-linear estimation and conditional simulation (CS) are alternative geostatistical approaches that can provide more reliable estimates of the recoverable tonnage and grade (i.e. the ultimate production grade–tonnage relationships) from wide-spaced drilling. Non-linear estimation and CS are not commonly used on iron ore deposits. However, these techniques have had wide application in other commodities such as gold and base metals. Conditional simulation has been used in the iron ore industry; however, its use is outlined in this paper as a means of generating non-linear estimates rather than for variability and drill spacing analysis. As a rule, regardless of commodity, the decision to use non-linear geostatistics will necessitate increased skills and require more time. This decision must therefore be justified in terms of cost and benefit. Such cost-benefit analysis is not straightforward, and to help, an approach for determining when linear estimates are inadequate is presented. The global ‘Discrete Gaussian Model’ (DGM) of change of support is a well established non-linear geostatistical approach. This method is recommended as a tool to establish whether progressing to non-linear methods will materially improve prediction of the ultimate grade–tonnage relationships. The additional use of DGM as a block model validation tool is also discussed. One specific factor contributing to the lack of application of non-linear geostatistical methods for iron ore deposits is the added difficulties that arise when CS and non-linear estimates are required to reproduce the numerous and often important correlations between variables. For this reason, a synoptic review of non-linear estimation and simulation methodologies applicable to correlated variables is presented.

Keywords: Iron ore, Non-linear estimation, Discrete Gaussian model, Uniform conditioning, Stepwise conditional transform, MAF

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List of abbreviations

CS	Conditional simulation
CCS	Conditional co-simulation
DGM	Discrete Gaussian model
ILC	Indirect lognormal correction
LC	Lognormal correction
MAF	Min./max. autocorrelation factors
MIK	Multiple Indicator kriging
OK	Ordinary kriging
OKCB	Ordinary kriging without minimising conditional bias
SCT	Stepwise conditional transform
SMU	Selective mining unit
UC	Uniform conditioning

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Introduction

This paper discusses the strengths and weaknesses of linear estimation (ordinary kriging, i.e. OK), non-linear estimation and conditional simulation (CS). These methods are examined with regard to their applicability to deposits such as iron ore where key variables are correlated and where such correlations must be honoured during estimation and simulation. This paper also introduces a method to determine whether progressing from linear to non-linear estimation or simulation will materially improve prediction of ultimate production grade–tonnage relationships.

It is now generally accepted practice for OK to be implemented as a means to produce minimum conditional bias linear estimates of grades (e.g. David, 1977; Krige, 1994, 1996a, 1996b; Matheron, 1963; Ravenscroft and Armstrong, 1990; Rivoirard, 1987). However, minimising conditional bias has the price of increased smoothing, a distortion of the

grade–tonnage curve that yields locally inaccurate predictions of the recoverable tonnes and grade above a cut-off grade. This distortion of the grade–tonnage curve is partly a function of the drilling density, but also depends on block size, search and variogram. Post-processing of kriged estimates can correct the grade–tonnage curve (see Krige *et al.*, 2004; Rossi and Parker, 1994). However, obtaining a single estimation map from wide spaced data relative to the selective mining unit (SMU) that is both globally minimising conditional bias and locally accurate at reproducing grade–tonnage relationships is mathematically impossible (see Caers, 2000; Journal *et al.*, 2000; Krige and Assibey-Bonsu, 2001). If linear kriging can be demonstrated as failing to adequately reproduce grade–tonnage relationships at final production selectivity, non-linear estimators (Vann and Guibal, 2000) or geostatistical CS (Lantuejoul, 2002) should be considered.

Non-linear methods have been used to estimate the proportion of small blocks or SMUs within large blocks or ‘panels’ that exceed a given cut-off grade. As well as non-linear kriging methods, e.g. uniform conditioning (UC) and multiple indicator kriging (MIK), non-linear estimates can also be obtained directly via CSs.

The first part of this paper deals with the motivations for non-linear methods, specifically the question of when such methods will yield practical improved inputs for mine planning. Since change of support lies at the heart of this issue, various approaches are reviewed, especially the discrete Gaussian model (DGM). This section concludes with a proposal for a methodology to determine when the employment of non-linear geostatistical methods for local resource estimation is warranted.

Introduced next is a case study from an iron ore deposit where the DGM was used in order to evaluate smoothing and selectivity.

Finally, a review is presented of various traditional and more recent approaches to applying non-linear geostatistical methods to multi-variable data sets, such as those commonly encountered in the iron industry.

Terms used in this paper:

- ‘Non-linear methods’ covers geostatistical methods where change of support is implemented or possible; both global and local. This includes both non-linear kriging *and* simulation approaches as well as DGM, but the term non-linear estimator denotes non-linear kriging unless otherwise specified in the text
- ‘Local estimation’ refers to estimation of block grades or grade distributions (i.e. grade–tonnage curves) to inform individual blocks in a block model
- ‘Global estimation’ denotes estimation of grades or grade distributions (i.e. grade–tonnage curves) of large subsets of a deposit, for example a domain. When grade–tonnage relationships are obtained, we say this is ‘global non-linear estimation’.

Assessing when non-linear methods will add value

Change of support

The need for a change of support model

The term ‘support’ denotes the volume upon which average values are computed or measured. The grades on smaller supports are said to be more dispersed (i.e. have higher variance) than grades on larger supports: this influence of support on variability is the ‘volume-variance’

or ‘support effect’ (Journal and Huijbregts, 1978). Grades of mineralisation measured on sample support can thus be much richer or poorer than grades measured on larger supports, say SMU blocks.

The tonnes and grade above cut-off based on sample support is usually a very poor predictor of tonnes and grade above cut-off obtained when making ore-waste decisions at SMU scale. This is especially so in the presence of high relative nugget effect or, equivalently, when the range of the variogram is short compared to drill sample spacing. To obtain acceptable grade–tonnage curves in these circumstances, it is necessary to derive the distribution of SMU values from the sample data distribution, and to do so a ‘change of support model’ (Emery and Torres, 2005) is needed. The key objectives for any change of support model are:

- The SMU and sample support distributions must have the same mean
- The variance of the SMU distribution must be that calculated by Krige’s Relationship (also called the ‘Volume Variance Relationship’). Krige’s Relationship states that the variance of blocks in a domain is equal to the variance of samples minus the within-block variance (which is equal to the average value of the variogram function within the SMU and can be calculated from the variogram model)
- Cartier’s Relationship must be satisfied, i.e.

$$E[Z(\underline{x})|Z(v)] = Z(v)$$

where \underline{x} denotes a point at random within the block v . In other words, if a random sample \underline{x} within a block v is to be obtained, the *expected* grade of this sample should be the block grade. Satisfaction of Cartier’s Relation ensures the preservation of grade–tonnage order-relationships; i.e. that the block distribution must be less selective than the sample distribution.

The discrete Gaussian model of change of support

The DGM for change of support is a well established approach and the basis of a widely used global estimation method for mining applications (Journal and Huijbregts, 1978; Vann and Sans, 1995). It is predicated on the idea that once the raw sample support distribution is transformed to a Gaussian (or normal) distribution, Krige’s relationship can be used to change this distribution to block support. The block support distribution is then back-transformed from Gaussian to obtain the raw block distribution. The key assumption for DGM is that the resulting transforms are *bivariate Gaussian* (i.e. Gaussian sample values and block values are linearly correlated). It is impossible to check this assumption of bi-Gaussianity; however, it *is* possible to check the plausibility of the assumption by running tests at sample level. If the bi-Gaussianity assumption is acceptable at sample level it is plausible that it is acceptable between samples and blocks. For more information on bi-Gaussianity refer to Chiles and Delfiner (1999) and Emery (2005).

The DGM, like all Gaussian based geostatistical methods, also relies on the underlying assumption that the spatial grade distribution can be best described by a ‘diffusion model’ where grade tends to move from lower to higher values and vice versa in a relatively continuous way (Rivoirard, 1994; Vann *et al.*, 2000). Another way of simply describing the diffusion model is to say that, when going

from higher to lower grade regions within a domain, there is a tendency to pass through intermediate grades.

An additional benefit of the DGM is that it can take into account the information effect, described in the next section.

The information effect

The final selection of SMUs in a selective mining operation is usually based on much denser information (production data) than was available at resource definition stage. However, this selection is still made on estimates since true SMU grades are not known. Although SMU estimates at grade control stage will be more precise than resource definition estimates, they will still be *estimates*. The 'information effect' describes the error and SMU misclassification that results from basing mining selection on SMU estimates rather than true values. To take information effect into consideration in resource estimation, the joint distribution of estimates and true grades for SMUs must be modelled. For further details on the information effect, see Journel and Huijbregts (1978) and Roth and Deraisme (2001).

Modelling information effect using discrete Gaussian model

The reduction in misclassification due to information effect can be modelled (Journel and Huijbregts, 1978, pp. 449–456). Modelling information effect using the DGM is dependent on the robustness of the variogram model at short scales, and also relies on the validity of the bi-Gaussianity assumption. Key kriging statistics for ultimate grade control (SMU) estimates, such as the kriging variance, can be calculated prior to mining, using the variogram model based on wider spaced drilling, and the assumed final production drilling configuration. It aids the robustness of this approach if at least some close-spaced drilling is available to infer the short-scale portion of the variogram accurately. In summary, it is possible via DGM to predict from wide-spaced resource definition drilling data the grade–tonnage curve of SMUs expected based on final production (grade control) estimates; i.e. at the time of mining.

Note that the assumption of final production (grade control) drill spacing, variograms and SMU size in operational iron ore environments should be relatively straightforward because the required parameters are either at hand, or can be reliably inferred. In a greenfields iron ore project, however, considerable care needs to be exercised in specifying these parameters and it is always prudent to consider sensitivity analysis for less reliably inferred parameters.

Free selection

The DGM assumes 'free selection' of SMUs within a panel or, in other words that all SMUs above the cut-off can be mined regardless of their relative locations. In reality, this is rarely completely true (for example, isolated blocks of ore may be dispatched with waste). However, free selection is a reasonable assumption in open pit mines because all SMUs within the ultimate pit shell will be mined irrespective of their grade.

Application of the discrete Gaussian model for the generation of global grade–tonnage curves

Until recently commercial mining and geostatistical software could only implement DGM as a means of univariate global estimation (i.e. estimation of global grade–tonnage curves for a single variable at a time).

This was a limitation for most iron ore deposits. If applied to more than one variable univariate DGM assumes independence of the variables; i.e. it ignored any correlations. However, if two variables are significantly correlated, the grade–tonnage curves are also related and independent application of the DGM to each variable will produce economically and technically erroneous results.

There is now commercially available geostatistical software (e.g. Isatis) that has bivariate and multivariate implementations of DGM, which has been applied to both global estimation of grade–tonnage curves and to local recoverable resource estimation (UC).

Applications of the discrete Gaussian model for ordinary kriging without minimising conditional bias (OKCB)

General practice (e.g. Krige 1994, 1996a, 1996b) has been to implement OK to obtain linear estimates with minimal conditional bias. This section describes an approach that has been proposed by Isaaks (2004) in which the number of samples used to estimate blocks is decreased to reduce smoothing, commonly until the global grade–tonnage curves from the kriged estimate match the equivalent curves generated from the DGM.

It has been argued by Isaaks (2004) that conditional bias is irrelevant to the accuracy of predicting the future recoverable resource when block estimates are used solely to estimate tonnes and grade for long-term mine design (e.g. pit optimisation). Isaaks concedes that kriged estimates used for selection at the time of mining (grade control) need to be conditionally unbiased.

Isaaks (2006) also provides an example in which blocks are deliberately estimated with low sample numbers, thus intentionally increasing conditional bias, with the objective of decreasing smoothing. He argues that local grade estimation accuracy decreases but the prediction of the *global* grade–tonnage relationships (specifically, global grade–tonnage curves) is improved. We call this approach here OKCB.

It is argued in this paper that local accuracy is of vital importance in resource estimate models used as a basis of long term design (pit design, locating pit walls, determining optimised pit shells), and for generating reliable schedules and sequencing; this point was demonstrated by Magri *et al.* (2003). In addition, overly restricted searches employed in the OKCB approach can even result in *global* bias when the data are clustered. This can be best seen through quantitative kriging neighbourhood analysis (see Vann *et al.*, 2003) where the mean grades can commonly vary significantly with different search strategies.

Locally, at the scale of panels, UC should produce more accurate estimates of recoverable tonnes and grade than OKCB. Despite this OKCB may be sufficiently accurate for some deposits (where change of support issues are not pronounced) and it could be argued that it is easier to understand, quicker to produce and easier to use than proper non-linear approaches. In cases where the geometry of domains is such that the larger blocks (or 'panels') used in UC or MIK are volumetrically unacceptable, OKCB may be an alternative, pragmatic choice for generating recoverable resource estimates that are at least globally acceptable. However, in such cases we argue that CS would produce superior results to UC, MIK or OKCB, and these methods are discussed further in a later section of this paper.

It should be borne in mind for the OKCB method that individual block estimates will be generally inaccurate and there is no useful measure of uncertainty. For example, what volume of grouped OKCB blocks provides an accurate measure of recoverable resources?

In summary, if drill spacing is wide and local predictions of grade–tonnage relationships are required, without conditional bias, then we argue that, in most circumstances, a local non-linear method is preferable to OKCB (see also McLennan and Deutsch, 2004).

Alternatives to the discrete Gaussian model of change of support

There are various approaches to change of support (Emery and Torres, 2005; Lantuejoul, 1988; Rossi and Parker, 1994).

Some proposed approaches other than DGM include:

- (i) affine correction,
- (ii) assumption of total de-skewing (i.e. assume that the blocks are normally distributed)
- (iii) the lognormal correction (LC) or the indirect lognormal correction (ILC) or 'lognormal shortcut'
- (iv) empirical approaches by CS.

All these global change of support models can also be (and have been) applied to local non-linear estimation to produce local recoverable resource estimates. They have the following main commonalities:

- (i) they leave the mean unchanged
- (ii) they apply a variance adjustment f , a factor between 0 and 1 by which the variance of samples is adjusted. This is calculated by the ratio of the block and sample standard deviations obtained from Krige's Relationship
- (iii) the resulting block distribution must be less selective (again, Cartier's Relation).

Ignoring change of support assumes that $f=1.0$ (i.e. the variance is unaffected by support) and this is untenable given the influence of support.

Affine correction

This approach is very simple (and simplistic). The affine correction assumes that f is a simple rescaling of the distribution by the ratio of standard deviation of blocks and that of samples (Isaaks and Srivastava, 1989). The standard deviation of the blocks is determined through Krige's Relationship (Journel and Huijbregts, 1978). Thus, for the Affine Correction, the shape of the distribution does not change; the variance reduces, but not relative skewness. Strictly, the affine correction can only be true for the case where sample grade distribution is symmetric, and it will perform very poorly when any significant skewness is present. In practice, in cases where grades have any significant nugget effect or short-scale structure evident on the variogram this approach is unacceptable.

Complete de-skewing

This is the antithesis of the affine correction in which normality (Gaussianity) of the block distribution is enforced; i.e. complete de-skewing. This approach assumes that the distribution of blocks follows a Gaussian distribution with a variance lower than the sample variance as predicted by Krige's Relationship. Complete de-skewing is only conceivable in an extreme situation where the grades on sample supports are for

practical purposes pure nugget (or noise) and is not, in the authors' view, a sensible method.

Lognormal correction

The LC assumes permanence of lognormality (Dowd, 1982, 1983). Permanence of Lognormality means that if the grades of samples are lognormal (or transformed to be so) then the grades of blocks are also lognormal. This permanence assumption can be shown to be theoretically unviable since the average of two lognormal random variables is not a lognormal random variable. However, permanence is a reasonable first order approximation if the sample distribution is *strictly* lognormal. Critically, LC does not necessarily preserve the mean, and this fault compounds as the sample distribution departs from lognormality. Note that although the assumptions of LC are not met in practice, it is more realistic than the Affine Correction, in that it allows de-skewing of the distribution.

LC is unacceptable in practice, and not generally used. As a result, the method was modified and called the 'indirect lognormal correction'.

Indirect lognormal correction

The ILC relies on a re-scaling of the results of LC (Isaaks and Srivastava, 1989). This re-scaling is by the ratio of the mean of values before LC divided by the mean of values after LC. This step ensures that the ILC preserves the mean, unlike LC. Again, in most practical mining applications, ILC has advantages over the affine correction because there is some de-skewing but this de-skewing is not fully controlled by the variogram.

Although used frequently as change of support model for MIK, the underlying assumption of permanence of lognormality is theoretically unsound and, in practice difficult to defend. This, added to the inferiority of the de-skewing compared to DGM leads it to be a second option for situations where DGM cannot be implemented.

Comparison of change of support models

All the change of support methods described above rely on assumptions; however, the assumptions of the DGM are less stringent and theoretically more valid. Case studies also demonstrate the DGM as superior to the alternative methods (Emery and Torres, 2005; Lantuejoul, 1988; Rossi and Parker, 1994) by the criteria of reproducing known grade distributions on larger supports. Vann and Sans (1995) provided an empirical test of DGM for a gold deposit in which degree of the de-skewing is highly accurate.

Empirical approaches by conditional simulation

Empirical approaches to the change of support have involved:

1. Using CS to obtain recoverable resource estimates and then factoring a linear kriged estimate (or an MIK estimate) to match these CSs (Rossi and Parker, 1994). The benefits of this method are that both the support effect and dilution can be incorporated using dig outlines that are potentially more realistic than using rectangular SMUs. The limitation of this method is that it is an order of magnitude more time consuming to implement than the other change of support methods. If the time is invested to create CSs it seems reasonable to use these simulations as a direct route to recoverable

resource estimation rather than to post-process a kriged estimate

2. Use of CSs to directly obtain an SMU distribution. This involves interpreting the frequency distribution of SMU-scale block values from n realisations of a simulation model as a grade tonnage curve on SMU support (McLennan and Deutsch, 2004).

Assessing when to move from linear to non-linear methods

End users of the model

Many mining engineers find non-linear resource models (however obtained) unsettling and more difficult to work with. This is because these models are not represented by a single grade estimate per variable and block, but rather provide an estimate of the grade-tonnage curve for SMU sized blocks within every panel. This has contributed to the current situation in which relatively few sites use such models routinely.

Abzalov (2006) has proposed an innovative method whereby the location of ore and waste sized SMUs (for UC) within a panel can be estimated; however, this method is not widely used.

Comparisons of discrete Gaussian model and linear kriging grade-tonnage curves

If grade-tonnage curves for a given domain, variable and SMU block size using both a linear estimator (in general OK) and the global DGM agree very closely, (what 'closely' means is discussed below) then it is argued here that the proposition that the OK estimates are locally suitable for mine planning could be accepted. Consequently, there would be no strong practical motivation for use of non-linear local recoverable resource estimation approaches like UC and MIK. In these cases, a linear kriging such as OK can form the basis of a realistic assessment of project economics.

However, if the DGM and OK grade-tonnage curves are *not* acceptably close, then there *is* motivation to consider local non-linear estimation such as UC. Another alternative would be to generate non-linear estimates directly using a CS approach as discussed by McLennan and Deutsch (2004).

Relative uncertainty and use of the model

It is necessary to consider relative uncertainties when assessing the results from comparison of global DGM grade-tonnage curves (which represent global recoverable resources) and those from linear kriging (e.g. OK). As an example, at a particular cut-off grade, we may find that the metal quantity for a linear estimate is within 5% of that predicted from the global DGM for a given domain. Given the accuracy of the estimation and the

global DGM this small difference may not be meaningful, in some circumstances it may be due to the relative precision of our inferences. Even if this 5% is a real difference, the data quality, geological domains, density, etc. may have much higher uncertainties. Alternatively, if the project is economically marginal, then a 5% difference in metal may be critical.

Finally, it should be understood that for studies that require reproduction of the grade variability at SMU support (e.g. bench height optimisation and blending) or multiple models of reality (e.g. risk analysis in pit optimisation), i.e. simulations are mandatory and linear or non-linear kriged models are unsuitable.

Resource classification and non-linear estimates

The JORC (2004) code and other similar codes do not currently specify confidence limits for Measured, Indicated and Inferred resources. Despite this, most Competent Persons have opinions about confidence limits and some companies have their own internal guidelines. These confidence limits are integral to assessing smoothing and the move from linear to non-linear methods.

Whereas JORC (2004) does not prohibit non-linear estimates to be classified as Measured, some mineral industry professionals believe that non-linear estimates cannot be classified as Measured; the basis of this belief seems to be that the location of ore and waste for SMU-sized blocks within panels is unknown in non-linear estimates and simulations. Those subscribing to this view are essentially saying that final production (grade control) data are required for Measured resource classification, which is a matter for the Competent Person to assess. If we accept this proposition, then Measured categorisation will always require final production (grade control) data *regardless of the estimate used*.

For any estimation method, the location of ore and waste has uncertainty. This uncertainty must be weighed against the scale and required accuracy of the proposed mining operation. Thus a 'blanket statement' that non-linear estimates cannot be classified as Measured seems unwise.

For example, BHP Billiton Iron Ore apply reconciliation tolerances to guide resource classification as shown in Table 1. The tolerance criteria are based on operational experience and are consistent with the approach to classification described in Yeates and Hodson (2006).

The discussion of the link between mining method and classification in Yeates and Hodson (2006) is particularly relevant to the discussion of whether or not a non-linear estimate can be considered as a Measured Resource. Although the decision to pursue a non-linear estimate implies a lack of knowledge of ore-waste

Table 1 BHP Billiton Iron Ore resource classification reconciliation tolerance guidelines

Resource class	Estimated component	3 Monthly reconciliation tolerance	12 Monthly reconciliation tolerance
Measured	Tonnage	± 15% relative difference	± 10% relative difference
	Fe%	± 1% absolute difference	± 0.5% absolute difference
	P, SiO ₂ , Al ₂ O ₃ , LOI%	± 15% relative difference	± 10% relative difference
Indicated	Tonnage	± 20% relative difference	± 15% relative difference
	Fe%	± 1% absolute difference	± 1% absolute difference
	P, SiO ₂ , Al ₂ O ₃ , LOI%	± 20% relative difference	± 15% relative difference
Inferred	Tonnage	± 30% relative difference	± 20% relative difference
	Fe%	± 1.5% absolute difference	+/- 1.5% absolute difference
	P, SiO ₂ , Al ₂ O ₃ , LOI%	± 30% relative difference	+/- 20% relative difference

boundary locations within a given (local) panel size, this may not be material for economically significant production periods.

The use of discrete Gaussian global grade–tonnage curves: BHP Billiton iron ore example

Resource evaluation procedures

At the concept and pre-feasibility level, significant resource uncertainty exists because reliable drill hole data are either absent or very broadly-spaced. Understanding the possible range of outcomes for poorly drilled deposits includes understanding the potentially recoverable resource as well as the global tonnage and grade ranges.

The DGM can be utilised for two purposes:

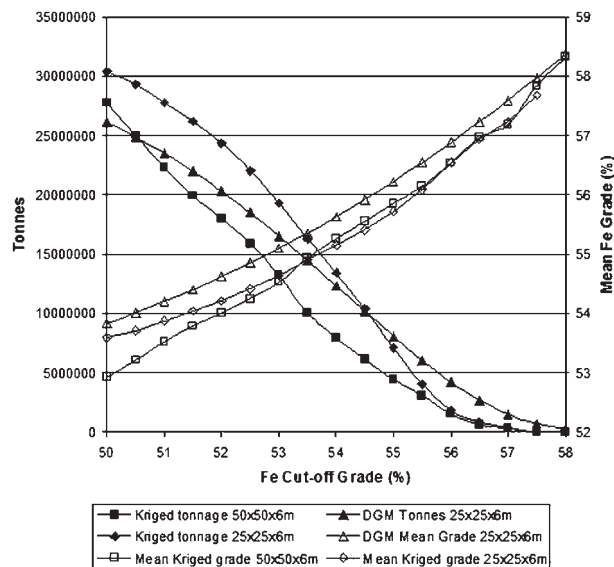
- (i) As a ‘validation tool’ to assess the degree of smoothing in kriged models. This is not proposed as an absolute validation but rather a process of comparison and assessment of potential risks:
 - a. DGM provides a relatively quick and cost-effective alternative estimation method, which can be compared to kriged estimates to understand potential risks in using the kriged estimate
 - b. DGM has been used to comparatively assess historical or legacy models, which were typically developed using sectional projection or inverse distance weighting interpolation methods
- (ii) To compare model selectivity to mining selectivity:
 - c. For bedded iron ore deposits, global continuity of geology and iron grade is generally very high and, as a result, drill spacing can be wide in comparison to the SMU. In some cases, drill spacing can approach a 50×50 m (X, Y) pattern compared to an SMU of $5 \times 5 \times 3$ m (X, Y, Z). Conversely, spatial continuity of contaminant grades (e.g. SiO_2 and Al_2O_3) can be very low
 - d. Some deposits have drill data that may be 600×100 m spaced or greater and therefore large kriging blocks are required to reduce conditional bias. This use of very large kriging blocks has the draw-back that it distorts the view of the ultimate production tonnage–grade relationships. Evaluation of the resource potential based on such coarse kriged models may result in misguided conclusions, particularly in a multi-pit blending scenario where cut-off grades in different pits could be critical in defining product characteristics.

In some cases, further study has been initiated when DGM results have showed a significantly different tonnage–grade relationship compared to the kriged model. Whilst absolute criteria have not been defined for triggering the use of various non-linear estimation methods, the assessment of DGM versus kriging has prompted pilot studies to understand the distribution of outcomes based on different mining selectivity scenarios as well as targeting other questions such as drill spacing and resource classification.

Results of discrete Gaussian global grade–tonnage curve analysis

A case study of iron grade–tonnage curves is presented here which considers three cases:

- (i) ‘DGM result’ for $25 \times 25 \times 6$ m blocks



1 DGM example for domain with high spatial variability of geology and geochemistry

- (ii) ‘Optimised kriging result’ based on $50 \times 50 \times 6$ m blocks (OK with optimised kriging neighbourhood and block size from quantitative kriging neighbourhood analysis)
- (iii) ‘Non-optimised kriging result’ for $25 \times 25 \times 6$ m blocks (as an example of a conditionally biased kriging result for a smaller block size).

Figure 1 considers a domain that exhibits a high degree of spatial variability in geology and geochemistry, which has a mixture of mineralised and non-mineralised material. Drill data is regularly spaced at 70×70 m. At a 54%Fe cut-off, the potentially recoverable resource tonnage estimated by DGM is significantly greater than estimated from the ‘optimised kriged’ model, which reflects the distortion of the grade–tonnage curve implicit in the linear kriging; i.e. the degree of smoothing in the kriged results is higher than acceptable. Kriging smaller blocks as a route to combat this distortion of the grade–tonnage curve produces a conditionally biased result where, at low cut-offs, the recoverable resource tonnage is greater than both the DGM and the ‘optimised kriging’, and at high cut-offs shows less recoverable resource compared to the DGM [Note that analysis based on DGM excludes mining dilution resulting from mining across domain boundaries. Experience in Pilbara iron ore deposits indicates that dilution can result in 80–95% recovery of tonnes and grade depending on local geology and mining scenarios]. This is clearly unacceptable.

DGM results in the case illustrated in Fig. 1 require further debate concerning resource classification. If it can be shown that the ‘optimised kriged’ model (the tool for public resource reporting in this instance) is likely to significantly under-predict the recoverable resource, then what is the appropriate classification for the domain given this uncertainty? There are two issues that must be considered by the Competent Person:

- (i) understanding geological and mining uncertainty
- (ii) publicly reporting resource estimates in accordance with Australian Stock Exchange listing rules.

Utilisation of grade–tonnage curve analysis by long-term planning

Grade–tonnage curve analysis by long-term planning engineers based on linear estimations is routinely carried out to understand recoverable resource scenarios at different cut-offs. Use of DGM as described above adds another dimension to the analysis and forces the debate about how data support and geological uncertainty relate to pit optimisation and design.

One of the main limitations of DGM work is that estimates are constrained within a domain and do not consider edge dilution. For assessing mining selectivity it is important to be able to incorporate edge dilution, particularly in iron ore deposits where very sharp transitions in grade can occur over very short distances due to stratigraphic controls on mineralisation.

Non-linear estimation and conditional simulation methods for correlated variables

Introduction

This section provides descriptions of estimation and simulation methodologies that are applicable for use with correlated variables and are thus of particular interest for obtaining non-linear estimates in the case of iron ore. Discussed below are UC and various implementations of CS. Multivariate implementations of MIK like estimators are plausible but to the authors' knowledge they have not been practically applied to resource estimates in the mining industry. These methods are not discussed further in this paper.

The multivariate non-linear kriging estimation discussed below is UC, arguably the only commercially operative practical option for multivariate non-linear kriging with change of support. Three methods are discussed that may be candidates for generation of non-linear resource estimates via a direct route using CS. Firstly, conditional co-simulation (CCS), which involves simulating while taking into account the correlation between variables. The remaining two simulation methods involve removing the correlation between variables, using min./max. autocorrelation factors (MAF) or the stepwise conditional transform (SCT) before performing independent simulation of the transformed variables.

Uniform conditioning

Description of uniform conditioning

The aim of non-linear kriging for recoverable resources is to estimate tonnes and grades assuming a certain SMU at panel scale. This is possible, but the location of ore and waste *within* panels remains unknown and will require later grade control. For a detailed technical description of UC and the underlying DGM the reader is referred to Guibal (1987), Rivoirard (1994) and Neufeld (2005).

The first step for a UC estimate is to produce linear (usually OK) estimates of panels that have minimised conditional bias. Uniform conditioning then involves deriving the local conditional distributions (conditional to the neighbouring information) of SMUs within these panels via the previously discussed DGM. Uniform conditioning can be implemented to account for the information effect.

Uniform conditioning should be implemented utilising panel sizes that take into account the following two criteria:

- (i) panels should be as small as possible, for geometric resolution, while still providing accurate estimates with minimal conditional bias
- (ii) panel size should be assessed in terms of SMU dimensions; i.e. the panels must be large enough to contain a reasonable number (generally >10) of SMUs because the *distribution* of SMUs within panels is being estimated.

Strengths and limitations of uniform conditioning

Uniform conditioning has been quite widely implemented in a commercial context and has generally proved robust. However, like all geostatistical tools it should be 'fit for purpose'. In this regard, some of the key issues regarding UC are:

- (i) domains with spatial grade distributions that cannot be reasonably described by a diffusion model are not suitable for UC
- (ii) as for MIK, UC can have order relations problems, i.e. tonnage and metal curves above grade cut-offs may not always be strictly decreasing mathematical functions, and grade above cut-off may not always be strictly an increasing function; however, these problems are easily corrected and (if the diffusion model holds) generally less severe than for indicator-based recoverable resource estimates.

If the above-mentioned diffusion assumption cannot be demonstrated to be acceptable then UC is not adapted to the problem because it uses the DGM change of support. However, the question of which change of support to use in this case is problematic because the alternatives (see Rivoirard, 1994 for a description of the alternatives) are not available in commercial software.

Uniform conditioning requires specialist software and relatively advanced geostatistical knowledge to understand and implement well. Although the method is more involved than OK it is an order of magnitude faster than CS and is based on a more consistent and reliable change of support model (DGM) than indicator-based approaches like MIK (Vann *et al.*, 2000).

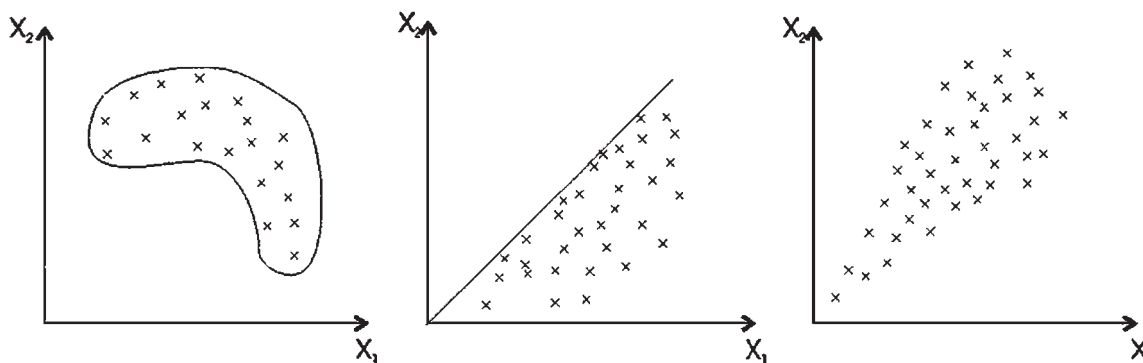
Recoverable resource estimates from UC, like all recoverable resource estimates, give grade–tonnage results per cut-off for each panel. This can be difficult for some mining engineers to use in pit optimisation but is not an insurmountable problem.

Conditional simulation

Conditional simulation

Geostatistical simulation is a spatial extension of the concept of Monte-Carlo simulation. In addition to reproducing the data histogram, geostatistical simulations also honour the spatial variability of data, usually characterised by a variogram model. If the simulations also honour the data values themselves, they are called 'conditional simulations'.

Geostatistical simulations generate a series of images, or 'realisations' as opposed to estimates, which output a unique result. A series of realisations presents a range of plausible possibilities about the spatial distribution of a variable that are consistent with the known statistical moments (variogram and histogram) and in the case of CS, the data themselves. Simulation thus has different



2 Examples of problematic bivariate distributions for geostatistical simulation: nonlinear relations (left), mineralogical constraints (centre), and heteroscedasticity (right), from Leuangthong and Deutsch (2003)

objectives to estimation. The point is to properly characterise and then reproduce the variance of the input data. If we have a sufficient set of realisations to describe the 'risk space', then geostatistical simulations provide an appropriate means to study problems relating to variability, for example risk analysis, in a way that estimates cannot. For example, simulations provide a path to evaluation and analysis of such issues as drill spacing, selectivity, blending, equipment selection, and sensitivity to different mine scheduling approaches (for more information refer to Humphreys and Shrivastava, 1997; Jackson *et al.*, 2003; Nicholas *et al.*, 2008; Sanguinetti *et al.*, 1994).

To date, the two most commonly used methods for CS in the mining industry are:

- (i) turning bands, which was the first large-scale 3D Gaussian simulation algorithm implemented (Journel, 1974)
- (ii) sequential Gaussian simulation (SGS), which is efficient and widely used (Lantuejoul, 2002).

Both of these simulation methods rely on the data having Gaussian distributions. Because in practice data never meet this criterion, the data must first be transformed into a Gaussian variable with a mean of zero and a standard deviation of one. After simulation of Gaussian values, the simulated nodes are back-transformed into a raw data distribution. Note that direct block simulation is also possible (Deraisme *et al.*, 2008).

CCS

Conditional simulation, as outlined above, deals with a single variable. If independent simulation of a multivariate dataset is performed (e.g. contaminants in an iron ore deposit), the resulting simulations will not reproduce any correlations between variables. Conditional co-simulation (CCS) generates realisations of more than one variable with the aim of preserving inter-variable correlations. Conditional co-simulation requires cross-variogram models for all pairs of variables as well as individual variogram models.

Strengths and limitations of conditional co-simulation

For CCS, experimental variograms and co-variograms are generated and modelled, noting that the resultant set is more difficult to fit with coherent models than independent experimental variograms. This is because a change to one of the variograms or cross-variogram models affects the fit of all the remaining models [The

constraint that variograms and cross-variogram models must be positive definite (Armstrong and Jabin, 1981) can sometimes be quite restrictive to obtaining models that are a good fit to the experimental variograms and cross-variograms].

Conditional co-simulation assumes, via the linear model of co-regionalisation, that correlations are indeed *linear*. Importantly, the linear model of co-regionalisation ensures that all the variances obtained from co-kriged estimates are zero or positive (Dowd, 1985; Shibli, 2003). In some datasets, the correlations between variables are not linear (Fig. 2) which is a limitation of CCS. We discuss alternatives in such cases later in this paper. Conditional co-simulation also assumes that the assumption of bi-Gaussianity is met.

Even after the set of variogram and co-variogram models are generated, CCS takes significantly more computer time than simulating the variables independently. Validation of CCS is also more time consuming because we must check that correlations are adequately reproduced. These complications have limited the use of CCS in mining applications generally, including in the iron ore industry, where the correlations between variables are usually very important.

Conditional simulation using the stepwise conditional transform (SCT)

Description of conditional simulation using the stepwise conditional transform

As described by Leuangthong and Deutsch (2003), this technique is implemented in the same way as a typical univariate CS with the exception of the SCT. This transformation has the objective of removing the correlation between variables. The variables can then be simulated independently, with the back-transformation of the simulations reinstating the correlation between the variables.

The order of the SCT of variables is important because the reproduction of the variogram and the correlations between variables (for the output simulations) will be most accurate for the first variable treated; with the reproduction quality decreasing for the second and subsequent variables. The process of simulation using SCT is discussed in more detail in the Appendix.

Strengths and limitations of conditional simulation using the stepwise conditional transform

The SCT process has the advantage of being relatively simple to implement and understand. Unlike

conventional CCS, the SCT removes all correlations between variables before simulation, making the technique better at handling the problematic correlations that are not well summarised by linear correlation coefficients (refer back to Fig. 2). The SCT approach thus makes modelling of experimental variograms and simulation faster than conventional CCS because cross variograms and co-kriging are not required.

Although the SCT removes any correlation at a lag of zero; there is no guarantee that variables are de-correlated at lags greater than zero. Consequently, before commencing simulation, the cross-variogram models of the transformed variables should be examined to confirm that there is no correlation between variables at lags greater than zero.

As discussed above, the reproduction of the variogram and the correlations between variables is best for the first, or primary, variable, and the reproduction quality decreases for the second variable and subsequent variables. This leads to an additional drawback of SCT, that there is no guarantee that modelled cross variograms will be reproduced in the realisations generated. Finally, the total number of samples in a domain practically limits the number of variables that can be transformed to two or three. Because of the stepwise process involved, the number of variables that can be transformed using the SCT process is practically limited by the number of samples that must be subjected to the transform. The more data we have in a domain, the more variables can be dealt with. Many iron ore deposits have a large number of domains resulting in many domains having relatively few samples, limiting the number of variables that can be considered to two or three. Since we need to estimate 5–6 or more variables in most practical situations this is a very real limitation of SCT for iron ore.

This would be a limitation for most iron deposits because many deposits have a large number of domains resulting in many domains having relatively few samples. This problem is further compounded because iron ore deposits typically have many correlated variables that must be simulated.

Unlike CCS, this method has not yet been widely used for mining applications and more case studies are required to build industry confidence in the method.

Conditional simulation using min./max. autocorrelation factors (MAF)

Description of conditional simulation using min./max. autocorrelation factors

Except for a factor transformation, MAF is implemented in the same way as a typical geostatistical Gaussian simulation (Desbarats and Dimitrakopoulos, 2000). The MAF transform removes the correlation between variables using an approach based on Principal Components Analysis and was developed by Switzer and Green (1984).

Principal Components Analysis is a multivariate statistical approach that aims to identify the least number of 'combined variables' (factors) that can explain a data set. For a more complete discussion of Factor Analysis, the reader is referred to Davis (1986) or Hair (1998).

Principal Components Analysis, like SCT, decorrelates variables that are 'co-located'; i.e. occur at the same

location. Variables at different locations are not necessarily decorrelated. For example, the cross-variogram of the factors for zero lag may show zero correlation but still reveal non-zero correlations for lags not equal to zero. In MAF, the de-correlation not only occurs at lag zero but also at a distance input by the user. This distance should be small and roughly correspond to the average sample spacing (Didier, pers. comm., 2006).

The process of simulation using MAF is summarised in the Appendix.

Strengths and limitations of conditional simulation using min./max. autocorrelation factors

The MAF transformation aims to remove correlations between variable elements before simulation. However, according to Rondon and Tran (2008), the MAF approach performs poorly with variables that do not demonstrate a *linear* correlation. This contrasts with the SCT approach, which is better equipped for handling problematic correlations represented in Fig. 2, including relationships not well captured by linear correlation coefficients.

The MAF de-correlation simplifies modelling of experimental variograms and makes simulation faster than conventional CCS because neither modelling of cross variograms nor co-kriging is required. On the other hand, the MAF transform (like the SCT approach) has not yet been widely used for mining applications.

The MAF transform is currently available in commercial software (e.g. Isatis).

Summary and conclusions

Linear estimation methods such as OK distort the grade–tonnage curves when only wide-spaced drilling is available. In general, even well implemented OK will have reduced variability because appropriate estimation blocks will be significantly larger than final (SMU) selection scale and because OK smoothes as a necessary price of minimising conditional bias. Direct estimation of SMU scale blocks is almost invariably unacceptable because the grade–tonnage curve is highly distorted, both globally and locally.

Non-linear estimation and CS are alternative approaches that can provide reliable estimates of the recoverable tonnage and grade from wide-spaced drilling. Local estimates can be obtained via these methods that conform to the theoretically expected grade–tonnage relationships at SMU scale.

A proposed way to assess the impact on grade–tonnage relationships, including smoothing, using geostatistical 'change of support' has been presented here. Specifically, a well-established non-linear geostatistical approach, the global DGM for change of support is recommended as a robust tool to establish when progressing from linear to non-linear estimates is warranted. In addition, the use of the DGM as a block model validation tool at BHP Billiton's iron operations has been described with a case study. Finally, a synoptic review of non-linear estimation and simulation methods that are applicable to correlated variables that could be suited to local recoverable resource estimation for iron ore or other multivariate deposits have been discussed.

While it is argued that quantifying the difference between grade–tonnage relationships for OK and DGM

is an important step in deciding when to move from linear to non-linear estimates, several additional factors must also be considered. First, the end users of the model (mine planning engineers) must be able to deal with the more complicated non-linear estimate or simulation. In part this is an education issue, but there are mechanical (data and file handling) aspects to be considered too. Second, the distortion of the grade-tonnage curve, including global degree of smoothing, must also be weighted against other potential inaccuracies in the model. Consequently, the stage of project evaluation (scoping versus pre-feasibility versus feasibility) and thus required accuracy must always be a framework for such considerations.

Isaaks (2004) proposes that in instances other than final, grade control estimation (where block estimation accuracy is self evidently critical), OK estimates that are deliberately conditionally biased (i.e. OKCB) can provide acceptable and accurate estimates of recoverable tonnes and grade above cut-offs. We have argued here that in most instances, local accuracy is not just a requirement for grade control – it is also important in medium to long term planning. When OK estimates are not appropriate inputs to mine planning, it is argued here that OKCB is the wrong solution and non-linear estimation or simulation is recommended.

In addition to non-linear kriging approaches to recoverable resource estimation such as UC and MIK, CS can provide a direct route to non-linear estimation, and thus recoverable resource estimation.

In iron ore, recoverable resource estimation may require consideration of two or more correlated variables. Multiple indicator kriging is not a practical industrial approach to multivariate recoverable resources. Uniform conditioning can be implemented for multivariate estimation. Conventional CCS, using the linear model of co-regionalisation, is robust and well tested and can account for the linear (or close to linear) correlations between variables; however, it is time consuming to implement and relies on the same bi-Gaussianity assumptions as UC. These issues have led to the development of methods such as MAF and SCT. These methods de-correlate variables thus allowing faster independent CS. Conditional co-simulation has received wide acceptance in the mining industry while MAF and SCT have some drawbacks and are yet to be practically demonstrated as viable alternatives.

In a case study of simulation methods of the Yandi Channel Iron Deposit in the Pilbara region of Western Australia, De-Vitry (2010) compared CCS, SCT and MAF. It was found that CCS (i.e. the linear model of coregionalisation) was significantly better at reproducing the characteristics of the declustered drill hole data, specifically the histograms, variograms, cross variograms and bivariate scatterplots. De-Vitry (2010) recommended that SCT only be attempted where significantly non-linear correlations are present and that MAF only be used where large numbers of correlated variables (e.g. >7) make conventional co-simulation difficult.

Acknowledgements

This paper is based on a previous paper (De-Vitry et al., 2007) covering the same subject matter but has been significantly re-written and updated. Scott Jackson is thanked for his input into the sections of that paper

outlining alternatives to the DGM and the section on CS and for his thorough review of the revised version of the paper appearing here. The original paper benefited from reviews by Andrew Bailey (BHP Billiton), Anthony Wesson (BHP Billiton), Ian Ashby (BHP Billiton) and Mike Job (Quantitative Group). Comments on that paper from one anonymous AusIMM reviewer were also helpful. The authors would finally like to thank BHP Billiton for permission to publish this work.

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Appendix

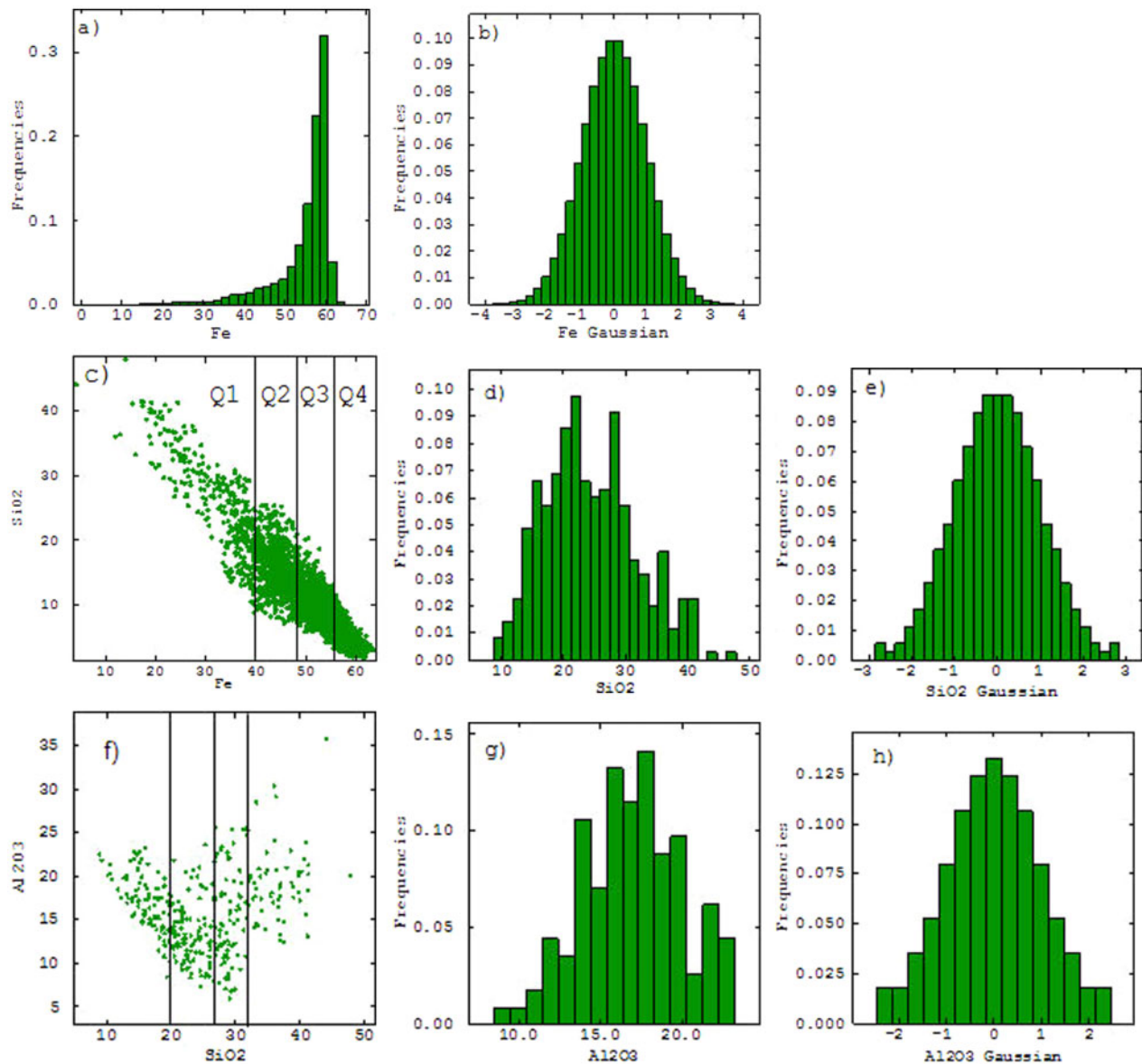
Description of simulation using the stepwise conditional transform

The steps for simulation using the SCT are (Leuangthong *et al.*, 2006):

- (i) decluster data to obtain representative distributions
- (ii) determine the order of variables for transformation
- (iii) apply the SCT to each variable, in the determined order, in order to remove the correlation between variables
- (iv) calculate and model experimental variograms of the transformed data
- (v) independently simulate the transformed variables
- (vi) back transform the simulated values by performing the SCT in reverse
- (vii) validate the simulation results.

The SCT transform is described below with an example where Fe is the primary variable, SiO₂ the secondary and Al₂O₃ the tertiary. Iron is transformed (Fig. 3a and b) simulated and back transformed in the same way as a standard CS as described above.

The transformation of SiO₂ is dependent (conditional) on the distribution of Fe. Take the scatterplot between Fe and SiO₂ represented in Fig. 3c where Fe is divided into quartiles. Leuangthong and Deutsch (2003) suggested that 10 classes are generally sufficient; however, in this case, for the purpose of illustration, quartiles have been chosen to simplify the graphical representation. The secondary variable (SiO₂) can then be classified according to Fe quartiles. All the SiO₂ samples within Fe quartile 1



3 An example of the SCT for Fe, SiO₂ and Al₂O₃: transformation of the raw primary variable *a* to Gaussian *b*; *c* dividing the primary variable into quartiles; transforming the SiO₂ distribution (Q1 *c*) from raw *d* to Gaussian *e*; *f* dividing the secondary variable into quartiles; and transforming the Al₂O₃ distribution (Q1 *f*) from raw *g* to Gaussian *h*

(Fig. 3c, Q1) can be transformed to a Gaussian distribution (Figs. 3d and 3e). This step is repeated for the SiO₂ samples within Fe quartile two and so on until there are four Gaussian distributions of SiO₂. These four distributions are then recombined to obtain a single Gaussian distribution. This Gaussian distribution of SiO₂ is uncorrelated to the Gaussian distribution for Fe. The variables transformed in this manner can thus be simulated independently.

To transform Al₂O₃ the samples from Fig. 3c quartile one are taken and the procedure described above is repeated (Figs. 3f, 3g and 3h). This is again repeated for quartiles two, three and four then all the Gaussian distributions (in this case 4×4=16 compared to 4 above for SiO₂) are recombined into a single Gaussian distribution. The key point here is that Al₂O₃ has fewer (compared to SiO₂) samples in each class when Gaussian transforms are made. Thus, the total number of samples for the primary variable limits the number of variables that can be transformed. Oy Leuangthong (pers. comm., 2006) recommends a minimum

of 10–20 samples per class, which will permit distribution inference, albeit a coarse and inaccurate one.

Description of simulation using MAF

Except for a factor transformation, this technique is implemented in the same way as a typical geostatistical Gaussian simulation (Desbarats and Dimitrakopoulos, 2000) that is:

- (i) decluster data to obtain representative distributions
- (ii) independently transform the raw variables to Gaussian
- (iii) perform the MAF transformation of the Gaussian variables in order to obtain the uncorrelated factors
- (iv) calculate and model experimental variograms of the transformed data
- (v) independently simulate the transformed variables
- (vi) back transform the simulated factors into Gaussian values
- (vii) back transform the Gaussian values into the raw variables
- (viii) validate the simulation results.