

A Guide to Selecting the Optimal Method of Resource Estimation for Multivariate Iron Ore Deposits

C De-Vitry¹, J Vann² and H Arvidson³

ABSTRACT

When only wide-spaced drilling is available, for example at concept, prefeasibility and feasibility stages, properly implemented linear estimation (including ordinary kriging) predicts grade tonnage relationships that are over-smoothed compared to final production estimates (and production). Non-linear estimation and conditional simulation are alternative geostatistical approaches that can provide more reliable estimates of the recoverable tonnage and grade from wide-spaced drilling.

Non-linear estimation and conditional simulation have been rarely used on iron ore deposits. However, these techniques have had wide application in other commodities such as gold and base metals. Conditional simulation has been used in the iron ore industry (eg Guibal *et al*, 1997); however, its use here is outlined as a means of generating non-linear estimates, not just its use for variability and drill spacing analysis.

As a rule, regardless of commodity, the decision to use non-linear geostatistics will necessitate increased skills and require more time. This decision must therefore be justified in terms of cost-benefit. Such cost-benefit analysis is not straightforward, and to help an approach for determining when linear estimates are inadequate is presented. Recommend in this paper is a well-established non-linear geostatistical approach, the global 'discrete Gaussian model' (DGM) of change of support, as a tool to establish whether moving from linear to non-linear estimates will materially improve estimate results. Also discussed is the additional use of DGM as a block model validation tool.

One specific factor contributing to the lack of application of non-linear geostatistical methods in iron is the added difficulties that arise when conditional simulation and non-linear estimates are required to reproduce the numerous and often important correlations between variables in iron ore deposits. Consequently, presented in the paper is a synoptic review of non-linear estimation and simulation methodologies applicable to correlated variables.

INTRODUCTION

Discussed in this paper are the strengths and weaknesses of linear estimation (ordinary kriging; OK), non-linear estimation and conditional simulation. These methods are examined with regard to their applicability to deposits such as iron ore where variables are correlated and these correlations must be honoured during estimation and simulation. Also introduced is a method for determining when to move from linear to non-linear estimation or simulation.

It is now generally accepted practice for OK to be implemented as a means to minimise conditional bias (eg David 1977; Krige, 1994, 1996a, 1996b; Matheron, 1963; Ravenscroft and Armstrong, 1990; Rivoirard, 1987). However, minimising conditional bias results in increased smoothing, yielding locally inaccurate predictions of the recoverable tonnes and grade above cut-off grade. The smoothing is partly a function of the drilling density, but also depends on block size, search and variogram. Post-processing of kriged estimates can correct for this smoothing

(see Krige, Assibey-Bonsu and Tolmay, 2004; Rossi and Parker, 1994). However, obtaining a single estimation map (from wide-spaced data relative to the SMU) that is both locally accurate, ie kriging and globally accurate, ie simulation in terms of recoverable tonnes above cut-off, is mathematically impossible (see Caers, 2000; Journal, Kyriakidis and Mao, 2000; Assibey-Bonsu, 2001). When smoothing is unacceptably high, non-linear estimators (Vann and Guibal, 2000) or geostatistical conditional simulation (Lantuejoul, 2002) should be considered.

Non-linear kriging methods attempt to estimate the proportion of small blocks or 'selective mining units' (SMUs) within large blocks or 'panels' that exceed a given cut-off grade. As well as a non-linear kriging methods (eg uniform conditioning and multiple indicator kriging), non-linear estimates can also be obtained via re-blocking of conditional simulations.

Discussed first in this paper is the need for non-linear methods, specifically when such methods will add value. Since change of support lies at the heart of this issue, various approaches are reviewed, especially the 'discrete Gaussian model' (DGM). Also described are the main methods for change of support and its implementation, both global and local. This section is concluded by proposing a methodology for determining when value will be added by utilising non-linear geostatistical methods.

Introduced next is a case study from an iron deposit in which the DGM was used as an adjunct to traditional approaches in order to evaluate smoothing and selectivity.

Discussed last is a review of various traditional and more recent approaches to applying non-linear geostatistical methods to multi-variable data sets, such as those commonly encountered in the iron industry.

Terms used in this paper:

- '*non-linear methods*' covers both non-linear kriging and simulation approaches, but the term non-linear estimator denotes non-linear kriging unless otherwise specified in the text;
- '*local estimation*' refers to estimation of block grades or grade distributions (ie grade tonnage curves) to inform a block model (ie 3D mapping of grade of blocks); and
- '*global estimation*' denotes estimation of grades or grade distributions (ie grade tonnage curves) of large subsets of a deposit, for example a domain.

ASSESSING WHEN NON-LINEAR METHODS WILL ADD VALUE

The discrete Gaussian model for change of support

The need for a change of support model

The term 'support' denotes the volume upon which average values are computed or measured. The grades on smaller supports are said to be more dispersed (higher variance) than grades on larger supports: this influence of support on variability is the 'volume-variance' or 'support effect' (Journal and Huijbregts, 1978). Grades of mineralisation measured on sample support can thus be much richer or poorer than grades measured on larger supports, say selective mining unit (SMU) blocks.

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1. Principal Consultant, Quantitative Group, PO Box 1304, Fremantle WA 6959. Email: cdv@qgroup.net.au
 2. Quantitative Group, PO Box 1304, Fremantle, WA 6959. Email: jv@qgroup.net.au
 3. BHP Billiton Iron Ore, 225 St Georges Terrace, Perth WA 6000. Email: heath.arvidson@bhpbilliton.com

The tonnes and grade above cut-off based on sample support is usually a very poor predictor of tonnes and grade above cut-off based on SMUs. This is especially so for increased relative nugget effect or, equivalently, if the range of the variogram is short compared to drill sample spacing. To obtain acceptable grade tonnage curves it is necessary to derive the distribution of SMU values from the sample data distribution, and to do so a 'change of support model' (Emery and Torres, 2005) is needed. The key objectives for any change of support model are:

- The SMU and sample support distributions must have the same mean.
- The variance of the SMU distribution must be that calculated by Krige's Relationship (also called the 'Volume Variance Relationship'). Krige's Relationship states that the variance of blocks in a domain is equal to the variance of samples minus the within-block variance (which is equal to the average value of the variogram function within the SMU and can be calculated from the variogram model).
- Cartier's Relation must be satisfied, ie:

$$E[Z(\underline{x})|Z(v)] = Z(v)$$

where:

\underline{x} denotes a point at random within the block v

In other words, if a random sample \underline{x} within a block v is obtained, the *expected* grade of this sample is the block grade. Satisfaction of Cartier's Relation ensures the preservation of grade-tonnage order-relationships, ie ensures that the block distribution must be less selective than the sample distribution.

The discrete Gaussian model of change of support

The discrete Gaussian model (DGM) of change of support is a well established approach and the basis of global estimation method for mining applications (Journel and Huijbregts, 1978; Vann and Sans, 1995). It is based on the idea that once the raw sample support distribution is transformed to a Gaussian distribution, Krige's relationship can be used to change this distribution to block support. The block support distribution is then back-transformed from Gaussian to the raw block distribution. The key assumption is that the resulting transforms are bivariate Gaussian (ie Gaussian sample values and block values are correlated). It is impossible to check this assumption of bi-Gaussianity; however, it is possible to check the plausibility of the assumption by running tests at sample level. If the bi-Gaussianity assumption holds at sample level it is plausible that it will hold between samples and blocks. For more information on bi-Gaussianity refer to Chiles and Delfiner (1999) and Emery (2005).

The DGM also relies on the underlying assumption that the spatial grade distribution can be best described by a 'diffusion model', where grade tends to move from lower to higher values and vice versa in a relatively continuous way (Vann, Guibal and Harley, 2000). Another way of simply describing the diffusion model is to say that when going from higher to lower grade regions within a domain, there is a tendency to pass through intermediate grades.

An additional benefit of the DGM is that it can take the information effect, described in the next section, into account.

The information effect

The final selection of SMUs will be based on much denser information (production data) than available at resource definition stage. However, this selection will still be made on estimates and, although these estimates will be more precise than resource definition estimates, they will still be *estimates*, ie not

the *true* grades of SMUs. This is the 'information effect' and, to take this into consideration, the joint distribution of estimates and true grades for SMUs must be modelled. For further details on the information effect, see Journel and Huijbregts (1978) and Roth and Deraisme (2001).

The resultant reduction in misclassification using the final production estimates due to information effect can be modelled (Journel and Huijbregts, 1978). Modelling information effect using the DGM is dependent on the robustness of the variogram model at short scales, and also relies on the validity of the bi-Gaussianity assumption. Key kriging statistics for ultimate grade control (SMU) estimates, such as the kriging variance, can be calculated prior to mining using the variogram model based on wider spaced drilling and the assumed final production drilling configuration. It aids the realism of this approach if at least some close-spaced drilling is available to infer the short-scale portion of the variogram. In summary, it is possible by utilising DGM with allowance for the information effect to predict from wide-spaced data of the grade-tonnage curve that will be obtained at the time of mining based on final production (or grade control) estimates.

Note that the assumption of final production (grade control) drill spacing, variograms and SMU size in operational iron ore environments should usually be relatively straightforward because the required parameters are either at hand, or can be reliably inferred. In a greenfield iron ore project, however, considerable care needs to be exercised in specifying these parameters and it is always prudent to consider sensitivity analysis for less reliably inferred parameters.

Free selection

The DGM assumes 'free selection' of SMUs within a panel or, in other words that all SMUs above the cut-off can be mined regardless of their relative locations. In reality, this is rarely completely true (for example, isolated blocks of ore may be dispatched with waste). However, free selection is a reasonable assumption in open pit mines because all SMUs within the ultimate pit shell will be dug up irrespective of their grade.

Applications of the discrete Gaussian model for the generation of global grade tonnage curves

Currently available software can only implement univariate DGM as a means of global estimation (ie estimation of global grade tonnage curves). Because of this, the DGM change of support can currently only be performed for one variable at a time without taking into consideration correlations between variables. If applied to more than one variable it assumes independence of the variables, ie neglects any correlations. However, if two variables are significantly (but not perfectly) correlated, the grade tonnage curves are also related (but not the same) and independent application of the DGM to each variable will produce economically and technically erroneous results.

In practice, for multivariate applications of the univariate DGM, the most economically important variable is selected for analysis. Alternatively, if time permits, the analysis can be performed on all the key variables independently.

Note: at the time of writing, via the M2RC Project, a bivariate implementation of DGM is under development and will eventually be available in Isatis™ software for application to both global estimation of grade tonnage curves and to local recoverable resource estimation (uniform conditioning). The M2RC Project work is being completed by Geovariances and Ecole Des Mines De Paris (Fontainebleau) and funded (at the time of writing) by Areva, BHP Billiton, Codelco, Lihir Gold, Newcrest and RioTinto.

Applications of the discrete Gaussian model for ordinary kriging without minimising conditional bias (OKCB)

General practice (eg Krige, 1994, 1996a, 1996b) is to implement OK with minimal conditional bias. This section outlines an approach proposed by Isaaks (2004) in which the number of samples used to estimate blocks is decreased to reduce smoothing, commonly until the global grade tonnage curves from the kriged estimate match the equivalent curves generated from the DGM.

Isaaks (2004) has argued that conditional bias is irrelevant to the accuracy of predicting future recoveries in the case where block estimates are not used for selection at the time of mining, but rather for the prediction of the tonnes and grade that will be used solely for long-term mine design (eg pit optimisation). Isaaks (2006) provides an example in which blocks are deliberately estimated with low sample numbers, thus intentionally increasing conditional bias with the objective of decreasing smoothing. He argues that local grade estimation accuracy decreases but the prediction of the *global* grade tonnage relationships, ie tonnes and grade above cut-off (ie global grade tonnage curves) is improved.

It is argued in this paper that local accuracy is important in resource estimate models for pit design (locating pit walls, determining optimised pit shells for example), and generating reliable schedules, sequencing, etc. This has been demonstrated by Magri *et al* (2003). In addition, overly restricted searches can result in global bias for clustered data. This can be best seen through quantitative kriging neighbourhood analysis (QKNA, see Vann, Jackson and Bertoli, 2003) where the mean grades can commonly vary significantly with different search strategies (ie numbers of samples). Conversely, huge searches should not be employed as this increases negative kriging weights and commonly results in no meaningful reduction in conditional bias.

Locally, ie at the scale of panels, uniform conditioning (UC) should produce more accurate estimates of recoverable tonnes and grade than OK with conditional bias (OKCB). Despite this OKCB may be sufficiently accurate for some deposits (where change of support issues are not pronounced) and it has the added benefit of being easier to understand, quicker to produce and easier to use. In the case where the geometry of domains is such that the larger blocks (or 'panels') used in UC or MIK are volumetrically unacceptable, OKCB may be a second, pragmatic choice for generating recoverable resource estimates that are at least globally acceptable.

However, it must be borne in mind that individual block estimates will be generally inaccurate with the OKCB method and there is no useful measure of uncertainty. For example, what volume of grouped OKCB blocks provides an accurate measure of recoverable resources? In summary, if drill spacing is wide and local predictions of grade tonnage relationships are required, without conditional bias, then we argue that, in most circumstances, a local non-linear method is preferable to OKCB (see also McLennan and Deutsch, 2004).

Alternatives to the discrete Gaussian model of change of support

There are various approaches to change of support (Emery and Torres, 2005; Lantuejoul, 1988; Rossi and Parker, 1994). One approach is to ignore this issue altogether. Some proposed approaches other than DGM include:

1. affine correction,
2. assumption of total de-skewing (ie assume that the blocks are normally distributed),
3. the lognormal correction or the indirect lognormal correction or 'lognormal shortcut', and
4. empirical approaches by conditional simulation.

All these global change of support models can also be (and have been) applied to local non-linear estimation, ie to produce local recoverable resource estimates. They have the following main commonalities:

1. They leave the mean unchanged.
2. They apply a variance adjustment f , a factor between zero and one by which the variance of samples is adjusted. This calculated by the ratio of the block and sample standard deviations.
3. The resulting block distribution must be less selective (again, Cartier's Relation).

Ignoring change of support assumes that $f = 1.0$ (ie the variance is unaffected by support), and this is untenable given the influence of support.

Affine correction

The affine correction assumes that f is a simple rescaling of the distribution by the ratio of standard deviation of blocks and that of samples. The standard deviation of the blocks is determined through Krige's relationship (Journel and Huijbregts, 1978).

Thus, for the affine correction, the shape of the distribution does not change, ie it reduces variance, but not relative skewness. Strictly, the affine correction is only true for the case where sample grade distribution is symmetric, and will perform very poorly in the case of variables for which there is a significant nugget effect or short-scale structure evident on the variogram. This approach is very simple (and simplistic).

Complete de-skewing

This is the antithesis of the affine correction: normality (Gaussianity) of the block distribution is assumed: ie complete de-skewing is effected. This approach assumes that the distribution of blocks is normal with lower variance. This is only reasonable if the blocks are very large with respect to the variogram range, ie when there is a very large variance reduction. Complete de-skewing is only conceivable in an extreme situation where the grades on sample supports are for practical purposes pure nugget (or noise).

Lognormal correction

The lognormal correction (LC), also called 'permanence of lognormality', assumes permanence of lognormality (Dowd, 1982, 1983), which can be shown to be theoretically unviable (the average of two lognormal random variables is not a lognormal random variable). However, permanence is a reasonable first order approximation if the sample distribution is *strictly* lognormal. Importantly, LC does not necessarily preserve the mean, and this fault compounds as the sample distribution departs from lognormality. Note that although the assumptions of LC are not met, in some ways it is more realistic than the affine correction, allowing de-skewing of the distribution.

LC is thus unacceptable in practice, and not generally used, leading to a modification – the 'indirect lognormal correction'.

Indirect lognormal correction

The indirect lognormal correction (ILC) relies on a re-scaling of the results of LC. This re-scaling is by the ratio of the mean of values before LC divided by the mean of values after LC. Importantly, by construction, ILC preserves the mean (unlike LC). Again, in most practical mining situations, ILC has advantages over affine because there is some de-skewing, but this de-skewing is not fully controlled by variogram.

Although used historically by practitioners of multiple indicator kriging (MIK) due to lack of alternative, the underlying

assumption of permanence of lognormality is theoretically unsound and, in practice, difficult to defend.

Comparison of change of support models

All the change of support methods described above rely on assumptions; however, the assumptions of the DGM are less stringent and theoretically more valid. Case studies also demonstrate the DGM as superior to the alternative methods (Emery and Torres, 2005; Lantuejoul, 1988; Rossi and Parker, 1994).

Empirical approaches by conditional simulation

Empirical approaches to the change of support involve utilising conditional simulations to calculate recoverable reserves and to factor a kriged estimate (or an MIK estimate) to match these conditional simulations (Rossi and Parker, 1994). The benefits of this method are that both the support effect and dilution can be incorporated using dig outlines that are potentially more realistic than using rectangular SMUs. The limitation of this method is that it is an order of magnitude more time-consuming to implement than the other change of support methods. If the time is invested to create conditional simulations why not use these simulations rather than factor a kriged estimate?

Assessing when to move from linear to non-linear methods

End users of the model

Many mining engineers find using non-linear resource models unsettling. This is because these models are not represented by a single grade estimate per variable and block, but rather provide an estimate of the grade tonnage curve for SMU sized blocks within every panel. This has contributed to the current situation in which relatively few sites use such models routinely. Discussed below are several solutions that could increase the use of non-linear resource models.

One method is training engineers in the benefits of non-linear models and how to use them. Another approach to making the result more user-friendly is to assign grades to SMU sized blocks within panels. Generally the location of ore and waste within panels is not known so the grades of SMUs are assigned at random but in the correct proportions. This assignment is arbitrary: a local recoverable resource estimation like UC (or MIK) yields estimates of proportions of SMUs above cut-off within a panel not the actual (located) SMU grades. Abzalov (2006) has proposed a method whereby the location of ore and waste-sized SMUs (for UC) within a panel can be estimated; however, this method is not widely used.

Comparisons of discrete Gaussian model and linear kriging grade tonnage curves

The aim is to generate grade tonnage curves for a given domain, variable and SMU block size using both a linear estimator (in general OK) and the global DGM. If the two agree very closely, (what 'closely' means is discussed below) then it could be accepted that the OK estimates are suitable for mine planning. If the two grade tonnage relationships do prove to be close, there is no strong practical motivation to move to non-linear local estimation: a linear kriging such as OK can form the basis of a realistic assessment of project economics.

However, if the DGM and OK grade tonnage curves are *not* acceptably close, then there is motivation to consider local non-linear estimation such as UC. Another alternative would be to generate non-linear estimates using a conditional simulation approach.

Relative uncertainty and use of the model

It is necessary to consider relative uncertainties when assessing the results from comparison of global DGM grade tonnage curves (which represent global recoverable resources) and those from linear kriging (eg OK). As an example, at a particular cut-off grade, the metal quantity for a linear estimate is within five per cent of that predicted from the global DGM for a given domain. Given the accuracy of the estimation and the global DGM this small difference may not be meaningful (ie may be due to the relative precision of our inferences). If this five per cent is in fact a real difference, then data quality, geological domains, density, etc may have much higher uncertainties, making issues of smoothing relatively minor.

Alternatively, if the project is economically marginal, then a five per cent difference in metal may be critical. Finally, for studies that require reproduction of the grade variability at sample support (eg bench height optimisation and blending) or multiple models of reality (eg risk analysis in pit optimisation) simulations are required.

Resource classification

The JORC (2004) code does not specify confidence limits for Measured, Indicated and Inferred resources. Despite this, most competent persons have some opinions about confidence limits and some companies have their own internal guidelines. These confidence limits are integral to assessing smoothing and the move from linear to non-linear methods.

While JORC (2004) does not prohibit non-linear estimates to be classified as Measured, some believe that non-linear estimates cannot be classified as Measured. This is because the location of ore and waste for SMU-sized blocks within panels is unknown in non-linear estimates and simulations. Those subscribing to this view are essentially saying that final production data are required for Measured resource classification, which is a matter for the Competent Person to assess.

For any estimation method, the location of ore and waste has some uncertainty. This uncertainty must be weighted against the size and required accuracy of the proposed mining operation. Thus, a statement that no non-linear estimates can be classified as Measured, while generally true, is not correct in all situations.

For example, BHP Billiton Iron Ore (BHPBIO) apply reconciliation tolerances to guide resource classification as shown in Table 1. The tolerance criteria are based on operational experience and are consistent with the approach to classification described in Yeates and Hodson (2006). Note that Table 1 in this paper forms only a part of the broader resource classification procedure within BHPBIO that also considers data quality, geological confidence and the other criteria as per JORC's Table 1 guideline.

The discussion of the link between mining method and classification in Yeates and Hodson (2006) is particularly relevant to the discussion of whether or not a non-linear estimate can be considered Measured. Although the decision to pursue a non-linear estimate implies a lack of knowledge of geological boundary locations within a given (local) panel size, this may not be material when considering the relationship between the panel size and a period of mine production that is significant in terms of cash flow.

Given the large tonnage movements for some BHPBIO operations it is conceivable that reconciliation of a non-linear estimate over a three-month period would produce results that satisfy the Measured classification criteria. However, in the BHP Billiton context a Measured classification prior to mining based on a non-linear estimate is not recommended.

TABLE 1
BHP Billiton Iron Ore resource classification reconciliation tolerance guidelines.

Resource class	Estimated component	Three-monthly reconciliation tolerance	Twelve-monthly reconciliation tolerance
Measured	Tonnage	± 15% relative difference	± 10% relative difference
	Fe %	± 1% absolute difference	± 0.5% absolute difference
	P, SiO ₂ , Al ₂ O ₃ , LOI %	± 15% relative difference	± 10% relative difference
Indicated	Tonnage	± 20% relative difference	± 15% relative difference
	Fe %	± 1% absolute difference	± 1% absolute difference
	P, SiO ₂ , Al ₂ O ₃ , LOI %	± 20% relative difference	± 15% relative difference
Inferred	Tonnage	± 30% relative difference	± 20% relative difference
	Fe %	± 1.5% absolute difference	± 1.5% absolute difference
	P, SiO ₂ , Al ₂ O ₃ , LOI %	± 30% relative difference	± 20% relative difference

THE USE OF DISCRETE GAUSSIAN GLOBAL GRADE TONNAGE CURVES AT BHP BILLITON IRON ORE

Resource evaluation procedures

BHP Billiton Iron Ore (BHPBIO) is currently in a phase of expansion supported by concept, prefeasibility and feasibility projects. At the concept and prefeasibility level, significant resource uncertainty exists because reliable drill hole data are either absent or very broadly spaced. Understanding the possible range of outcomes for poorly drilled deposits includes understanding the potentially recoverable resource as well as the global tonnage and grade ranges.

BHPBIO's standard method of resource estimation is via OK into block models. In recent years, BHPBIO has realised the limitations of kriged models in the context of broadly-spaced drill data, and have utilised DGM for two purposes:

- As a validation tool to assess the degree of smoothing in kriged models. This is not proposed as an absolute validation but rather a process of comparison and assessment of potential risks:
 - DGM provides a relatively quick and cost-effective alternative estimation method, which can be compared to kriged estimates to understand potential risks in using the kriged estimate.
 - DGM has been used to assess historical or legacy models, which were typically developed using sectional projection or inverse distance weighting interpolation methods. These methods are considered inadequate in terms of current BHP Billiton standards.
- To compare model selectivity to mining selectivity:
 - For bedded iron ore deposits, global continuity of geology and iron grade is very high and, as a result, drill spacing is very broad in comparison to the SMU. In some cases, drill spacing can approach a 50 m × 50 m (X, Y) pattern compared to an SMU of 5 m × 5 m × 3 m (X, Y, Z). Spatial continuity of contaminant grades (eg SiO₂ and Al₂O₃) can be very low.

- Some deposits have drill data that may be 600 m × 100 m spaced or greater and therefore extremely large kriging blocks are required to avoid conditional bias, which distorts the view of the tonnage-grade relationship. Evaluation of the resource potential based on such coarse kriged models may result in misguided conclusions, particularly in a multi-pit blending scenario where cut-off grades in different pits could be critical in defining product characteristics.

BHPBIO resource modelling procedures now include DGM as a standard validation tool for the reasons described above. In some cases, further study has been initiated when DGM results have showed a significantly different tonnage-grade relationship compared to the kriged model. Whilst absolute criteria have not been defined for triggering the use of various non-linear estimation methods, the assessment of DGM versus kriging has prompted pilot studies to understand the distribution of outcomes based on variable mining selectivity scenarios as well as targeting other questions such as drill spacing and resource classification.

Results of discrete Gaussian global grade tonnage curve analysis

A case study of three grade-tonnage curves is presented here (Figure 1), which considers three cases:

- 'DGM result' for 25 × 25 × 6 m blocks,
- 'optimised kriging result' based on 50 × 50 × 6 m blocks (OK with optimised kriging neighbourhood and block size), and
- 'non-optimised kriging result' for 25 × 25 × 6 m blocks (as an example of a conditionally biased kriging result for a smaller block size).

Figure 1 considers a domain that exhibits a high degree of spatial variability in geology and geochemistry, which has a mixture of mineralised and non-mineralised material. Drill data is regularly spaced at 70 × 70 m. At a 54 per cent Fe cut-off, the potentially recoverable resource tonnage estimated by DGM is significantly greater than estimated from the 'optimised kriged' model, which reflects the mixed nature of the domain and the degree of smoothing in the kriged results. Kriging smaller blocks produces a conditionally biased result where at low cut-offs the recoverable resource tonnage is greater than both the DGM and the 'optimised kriging', and at high cut-offs shows less recoverable resource compared to the DGM.

Note that analysis based on DGM excludes mining dilution resulting from mining across domain boundaries. Experience in iron ore indicates that dilution can result in 80 - 95 per cent recovery of tonnes and grade depending on local geology and mining scenarios.

DGM results in the case illustrated in Figure 1 prompts debate concerning resource classification. If it can be shown that the 'optimised kriged' model (the tool for public resource reporting) is likely to significantly under-predict the recoverable resource, then what is the appropriate classification for the domain given this uncertainty? There are two issues involved:

- understanding geological and mining uncertainty, and
- publicly reporting resource estimates in accordance with stock exchange listing rules.

Geological and mining uncertainty, in the context of the entire deposit, is considered as low, given that there is potential upside to resource recovery rather than downside. It is also considered that opportunity to recover additional resource within the proposed pit design exists, given the in-pit grade control sampling strategy.

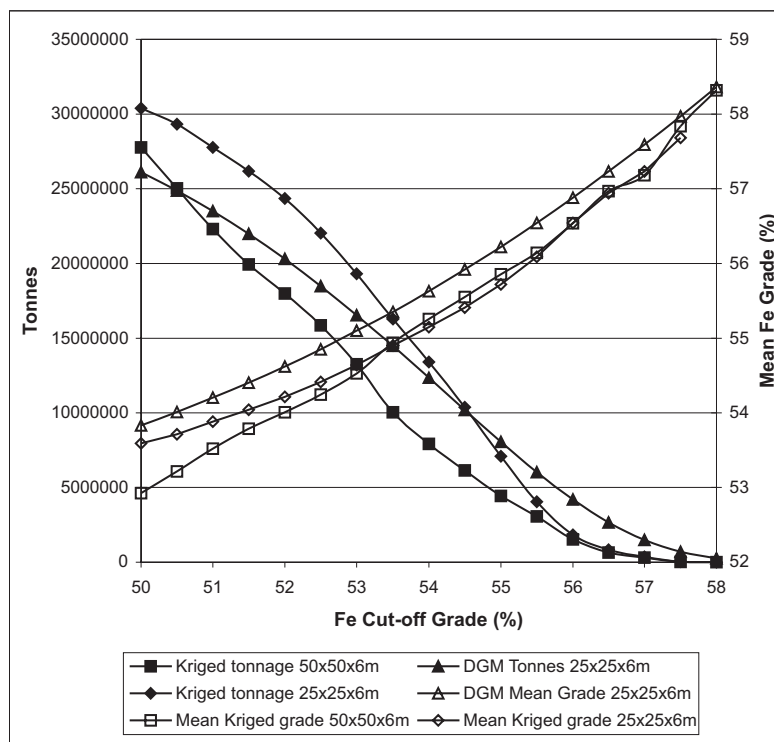


FIG 1 - Discrete Gaussian model (DGM) example for domain with high spatial variability of geology and geochemistry.

The view adopted for public reporting is that kriged blocks above the resource cut-off grade can be classified with confidence because the DGM results show there is a high degree of certainty that the resource at that cut-off will be recovered. The potential upside is not of a magnitude that was considered material for public reporting.

Utilisation of grade tonnage curve analysis by long-term planning

Grade tonnage curve analysis by long-term planning engineers based on linear estimations is routinely carried out for BHPBIO deposits to understand recoverable resource scenarios at different cut-offs. Use of DGM as described above adds another dimension to the analysis and forces the debate about how data support and geological uncertainty relate to pit optimisation and design.

One of the main limitations of DGM work is that estimates are constrained within a domain and do not consider edge dilution. For assessing mining selectivity it is important to be able to incorporate edge dilution, particularly in iron ore deposits where very sharp transitions in grade can occur over very short distances due to stratigraphic controls on mineralisation.

The ability to consider multivariate scenarios via DGM would add significant value to the processes and studies described above. Assessment of phosphorous, silica and alumina grades given a particular iron grade cut-off would provide additional support for mining selectivity decisions.

NON-LINEAR ESTIMATION AND CONDITIONAL SIMULATION FOR CORRELATED VARIABLES

Introduction

This section provides descriptions of estimation and simulation methodologies that are applicable for use with correlated variables and are thus of particular interest for iron ore. Discussed below are uniform conditioning (UC) and various implementations of

conditional simulation. Multivariate implementations of MIK-like estimators are plausible but to our knowledge they have not been applied to resource estimates in the mining industry. These methods are not discussed further in this paper.

The two estimation methods discussed are implementation of OK without minimising conditional bias (herein referred to as OKCB) and UC. Three conditional simulation methods are discussed. Firstly, conditional co-simulation (CSS), which involves simulating while taking into account the correlation between variables. The remaining two simulation methods involve removing the correlation between variables (using min/max autocorrelation factors (MAF) or the stepwise conditional transform (SCT) before performing independent simulation of the transformed variables.

Uniform conditioning

Description of uniform conditioning

For a detailed technical description of UC, and the underlying DGM the reader is referred to Guibal (1987), Rivoirard (1994) and Neufeld (2005).

The first step for a UC estimate is to produce linear (usually OK) estimates of panels that have minimised conditional bias. UC aims at deriving the local conditional distributions (conditional to the neighbouring information) of SMUs within these panels via the previously discussed DGM. Uniform conditioning can be implemented to account for the information effect.

UC utilises panel sizes that are as small as possible while still providing accurate estimates with minimal conditional bias. This effectively communicates uncertainty by indicating confidence in the estimates of tonnes and grades at panel scale, but the location of ore and waste *within* panels is unknown. There is also a consideration of panel size in terms of SMU dimensions, ie these panels must be large enough to contain a reasonable number (>10) of SMUs because the distribution of SMUs within panels is being estimated.

Strengths and limitations of uniform conditioning

Some of the key issues regarding UC are:

1. at present the public domain software implementations of UC are univariate;
2. domains with spatial grade distributions that cannot be best described by a diffusion model are not suitable for UC; and
3. UC can have order relations problems, ie tonnage and metal curves above grade cut-offs should be decreasing mathematical functions and grade an increasing function; however, these problems are easily corrected and (if the diffusion model holds) generally less severe than for indicator-based recoverable resource estimates.

If the above-mentioned diffusion assumption is not acceptable then UC is not adapted to the problem because it uses the DGM change of support. However, the question of which change of support to use in this case is problematic because the alternatives (see Rivoirard, 1994, for a description of the alternatives) are not available in commercial software.

UC requires specialist software and relatively advanced geostatistical knowledge to understand and implement well. Although the method is more involved than OK it is an order of magnitude quicker than conditional simulation and is based on a more consistent and reliable change of support model (DGM) than indicator-based approaches like MIK (Vann, Guibal and Harley, 2000).

Recoverable resource estimates from UC, like all recoverable resource estimates, give grade-tonnage results per cut off for each panel. This can be difficult for some mining engineers to use in pit optimisation, but is not an insurmountable problem.

Conditional simulation

Conditional simulation

Geostatistical simulation is a spatial extension of the concept of Monte Carlo simulation. In addition to reproducing the data histogram, geostatistical simulations also honour the spatial variability of data, usually characterised by a variogram model. If the simulations also honour the data values themselves, they are called 'conditional simulations'.

Geostatistical simulations generate a series of images, or 'realisations' as opposed to estimates, which output a unique result. A series of realisations presents a range of plausible possibilities about the spatial distribution of a variable that are consistent with the known statistical moments (variogram and histogram) and in the case of conditional simulations, the data themselves. Simulation thus has different objectives to estimation. The point is to properly characterise and then reproduce the variance of the input data. Geostatistical simulations provide an appropriate means to study problems relating to variability, for example risk analysis, in a way that estimates cannot. For example, simulations provide a path to evaluation and analysis of such issues as drill spacing, selectivity, blending, equipment selection and sensitivity to different mine scheduling approaches (for more information refer to Humphreys and Shrivastava, 1997; Jackson *et al*, 2003; Sanguinetti *et al*, 1994).

To date, the two most commonly used methods for conditional simulation in the mining industry are:

1. Turning bands (TB), which was the first large-scale 3D Gaussian simulation algorithm implemented (Journel, 1974). The method works by simulating one-dimensional processes on lines regularly spaced in 3D.
2. Sequential Gaussian simulation (SGS), which is efficient and widely used (Lantuejoul, 2002). This algorithm defines

a random path through all grid nodes (including the conditioning samples). Simple kriging (SK) of the nodes in the path occurs generating a local conditional distribution. A new value drawn from this local distribution and added to the nodes in the random path and the next node simulated, and so on.

Both of these simulation methods rely on the data having Gaussian distributions. Because in practice data never meets this criterion, the data must first be transformed into a Gaussian variable with a mean of zero and a standard deviation of one. After simulation of Gaussian values, the simulated nodes are back-transformed into a raw data distribution.

Conditional co-simulation

Conditional simulation, as outlined above, deals with a single variable. If independent simulation of a multivariate data set is performed (eg contaminants in an iron ore deposit), the resulting simulations will not reproduce any correlations between variables. Conditional co-simulation (CCS) is conditional simulation of more than one variable where inter-variable correlations are preserved in the output realisations. Conditional co-simulation utilises co-variogram models of the variables as well as individual variogram models to characterise the spatial correlation of and between *all* the variables.

Strengths and limitations of conditional co-simulation

For CCS, experimental variograms and co-variograms are generated and modelled. The experimental variograms and co-variograms are more difficult to fit with coherent models than independent experimental variograms. This is because a change to one of the variograms or co-variograms models affects the fit of all the remaining models. The constraint that variograms and co-variograms models must be positive definite (Armstrong and Jabin, 1981) is also quite restrictive to obtaining models that are a good fit to the experimental variograms and co-variograms.

Conditional co-simulation assumes, via the linear model of co-regionalisation, that correlations are linear. The linear model of co-regionalisation is a method used to ensure that all the variances obtained from co-kriged estimates are zero or positive (Dowd, 1985; Shibli, 2003). In some data sets, the correlations between variables are not linear (Figure 2), which is a limitation of CCS.

Once variogram and co-variogram models are generated, the co-simulations take significantly more computer time than simulating the variables independently. Validation of the simulations is also more time consuming because correlations must be adequately reproduced. These complications have limited the use of simulation in the iron ore industry, where the correlations between variables are usually very important, and have led to the development of the alternatives discussed below. Conversely, CCS is well tested for mining applications.

Conditional simulation using the stepwise conditional transform

Description of conditional simulation using the stepwise conditional transform

As described by Leuangthong and Deutsch (2003), this technique is implemented in the same way as a typical univariate conditional simulation with the exception of the stepwise conditional transform (SCT). This transformation has the objective of removing the correlation between variables. This allows the variables to be simulated independently, with the back-transformation of the simulations reinstating the correlation between the variables.

The order of the SCT of variables is important because the reproduction of the variogram and the correlations between

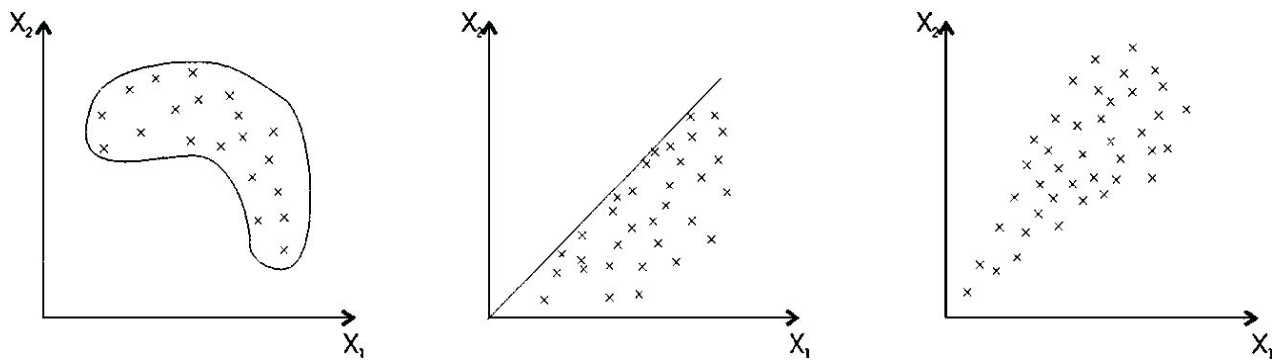


FIG 2 - Examples of problematic bivariate distributions for geostatistical simulation: non-linear relations (left), mineralogical constraints (centre), and heteroscedasticity (right), from Leuangthong and Deutsch, 2003.

variables (for the output simulations) will be most accurate for the primary variable and the reproduction quality decreases for the second variable and subsequent variables.

The steps as described by Leuangthong *et al* (2006) are:

1. decluster data to obtain representative distributions;
2. determine the order of variables for transformation;
3. apply the SCT to each variable, in the determined order, in order to remove the correlation between variables;
4. calculate and model experimental variograms of the transformed data;
5. independently simulate the transformed variables;
6. back-transform the simulated values by performing the SCT in reverse; and
7. validate the simulation results.

The SCT transform is described below, with an example where Fe is the primary variable, SiO₂ the secondary and Al₂O₃ the tertiary. Iron is transformed (Figures 3a and 3b) simulated and back-transformed in the same way as a standard CS as described above.

The transformation of SiO₂ is dependant (conditional) on the distribution of Fe. Take the scatterplot between Fe and SiO₂ represented in Figure 3c where Fe is divided into quartiles. Leuangthong and Deutsch (2003) suggest that ten classes are generally sufficient; however, in this case, for the purpose of illustration, quartiles have been chosen to simplify the graphical representation. The secondary variable (SiO₂) can then be classified according to Fe quartiles. All the SiO₂ samples within Fe quartile 1 (Figure 3c, Q1) can be transformed to a Gaussian distribution (Figures 3d and 4e). This step is repeated for the SiO₂ samples within Fe quartile 2 and so on until there are four Gaussian distributions of SiO₂. These four distributions are then recombined to obtain a single Gaussian distribution. This Gaussian distribution of SiO₂ is uncorrelated to the Gaussian distribution for Fe. The variables transformed in this manner can thus be simulated independently.

To transform Al₂O₃ the samples from Figure 3c quartile 1 are taken and the procedure described above is repeated (Figures 3f, 4g and 4g). This is again repeated for quartiles 2, 3 and 4, then all the Gaussian distributions (in this case 4 × 4 = 16, compared to four above for SiO₂) are recombined into a single Gaussian distribution. The key point here is that Al₂O₃ has fewer (compared to SiO₂) samples in each class when Gaussian transforms are made. Thus, the total number of samples for the primary variable limits the number of variables that can be transformed. Leuangthong (2006) recommends a minimum of 10 - 20 samples per class, which will permit distribution inference, albeit a coarse and inaccurate one.

Strengths and limitations of conditional simulation using the stepwise conditional transform

The SCT process, whereby correlations are removed, has the advantage of being relatively simple to implement and understand. Unlike conventional simulation, the SCT removes all correlations between variables before simulation, making the technique better at handling the problematic correlations that are not well summarised by correlation coefficients (refer back to Figure 2). This approach (de-correlation) also makes modelling of experimental variograms and simulation faster than conventional CCS (ie because cross variograms and co-kriging are not required).

The SCT removes any correlation at a lag of zero; however, there is no guarantee that variables are de-correlated at lags greater than zero. Consequently, before commencing simulation, the co-variogram models of the transformed variables need to be examined to determine that there is no correlation between variables at lags greater than zero.

As discussed above, the reproduction of the variogram and the correlations between variables is best for the primary variable, and the reproduction quality decreases for the second variable and subsequent variables. An additional drawback of the method is that there is no guarantee that cross variograms will be reproduced. Finally, the total number of samples in a domain limits the number of variables that can be transformed. This would be a limitation for many iron deposits which, although large, have numerous domains which results in many domains having few samples. This problem is further compounded because iron ore deposits typically have many correlated variables that must be simulated.

Unlike CSS, this method has not yet been widely used for mining applications and more case studies are required to build industry confidence in the method.

Conditional simulation using min/max autocorrelation factors

Description of conditional simulation using min/max autocorrelation factors

Except for the factor transformation, this technique is implemented in the same way as a typical geostatistical Gaussian simulation (Desbarats and Dimitrakopoulos, 2000). That is:

1. decluster data to obtain representative distributions,
2. independently transform the raw variables to Gaussian,
3. perform the min/max autocorrelation factors (MAF) transformation of the Gaussian variables in order to obtain the uncorrelated factors,

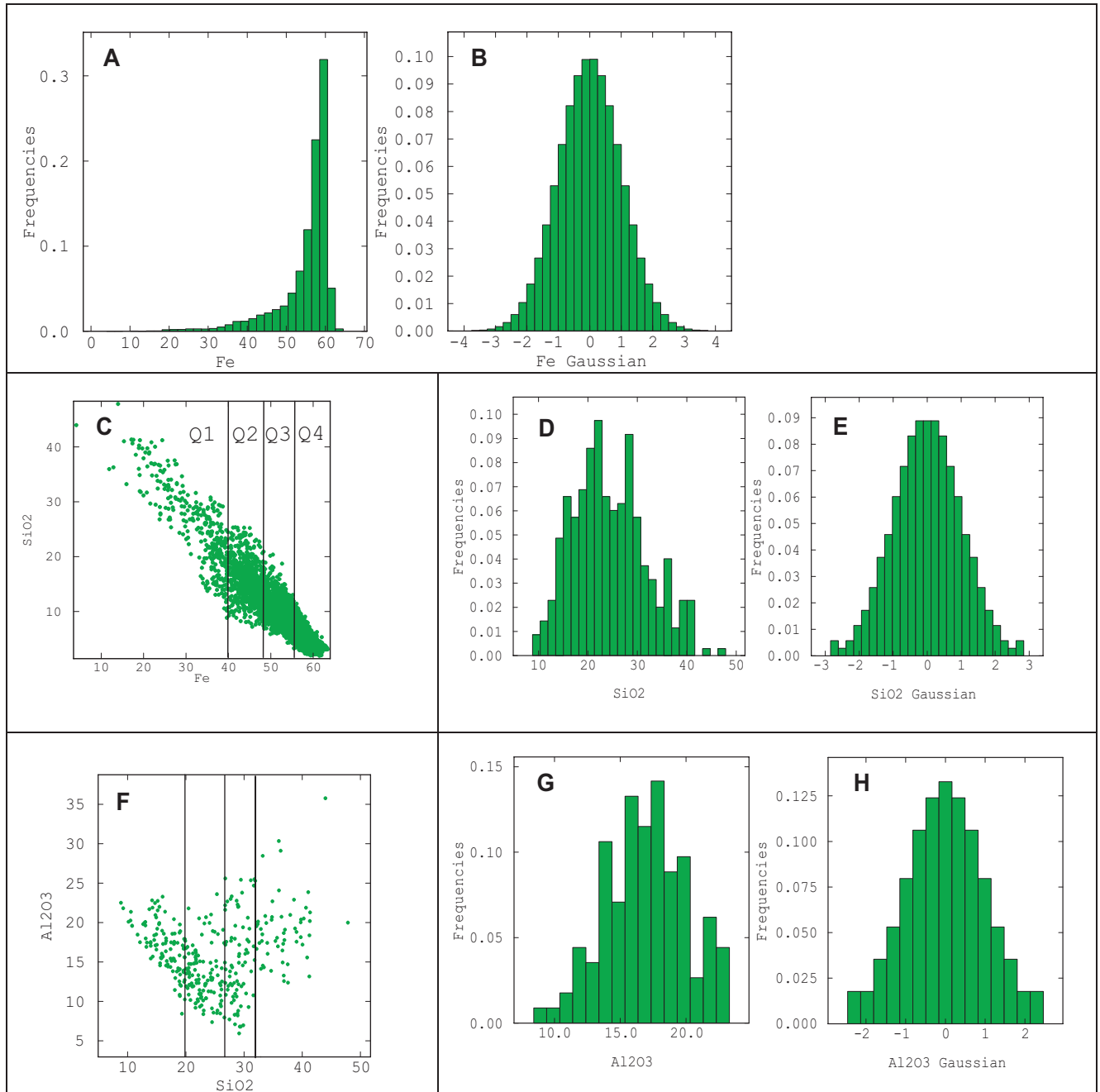


FIG 3 - An example of the stepwise conditional transform (SCT) for Fe, SiO₂ and Al₂O₃: Transformation of the raw primary variable (A), to Gaussian (B); (C) dividing the primary variable into quartiles; transforming the SiO₂ distribution (C) Q1; from raw (D), to Gaussian (E), (F) dividing the secondary variable into quartiles; and transforming the Al₂O₃ distribution (F) Q1; from raw (G) to Gaussian (H).

4. calculate and model experimental variograms of the transformed data,
5. independently simulate the transformed variables,
6. back-transform the simulated factors into Gaussian values,
7. back-transform the Gaussian values into the raw variables, and
8. validate the simulation results.

The MAF transform removes the correlation between variables using an approach based on principal components analysis (PCA). The MAF transformation process was developed by Switzer and Green (1984). For a comprehensive theoretical

coverage of factor analysis, the reader is referred to Davis (1986) or Hair *et al* (1998).

PCA is an exploratory statistical approach which has the objective to take a large set of (possibly) correlated variables and postulate a smaller set of variables (factors) that account for a (hopefully large) proportion of the multivariate variability. PCA factors quantify 'grouped correlations' and thus indicate which variables:

1. belong together, or, equivalently; and
2. seem to measure the same thing (ie the factors identify redundancy).

PCA aims to identify the least number of 'combined variables' (factors) that can explain the data set.

PCA and the SCT de-correlate variables that occur at the same location. Variables at different locations are not necessarily de-correlated. For example, at a lag of zero a cross variogram may show zero correlation but at a lag of 50 m variables may be correlated. In MAF the de-correlation not only occurs at lag zero but also at a distance input by the user. This distance should be small and roughly correspond to the data sampling (Didier, 2006). According to Vargus-Guzman and Dimitrakopoulos (2003) when the variables estimated can be represented by a two-structure linear model of co-regionalisation, the MAF procedure de-correlates the transformed variables at all lags, not just the distance input by the user. If there are for example five variables, five factors are generated. However, each factor is a different linear combination of the five variables.

Strengths and Limitations of conditional simulation using min/max autocorrelation factors

Unlike conventional simulation, the MAF transformation removes all correlations between variable elements before simulation. As for the SCT approach, this better equips the technique for handling problematic correlations represented in Figure 2. This de-correlation simplifies modelling of experimental variograms and makes simulation faster than conventional CCS (ie because cross variograms and co-kriging are not required). Conversely, unlike CSS the MAF transform (like the SCT approach) has not yet been widely used for mining applications.

According to Desbarats and Dimitrakopoulos (2000) for a two-structure linear model of co-regionalisation, the min/max autocorrelation factors are ranked in order of increasing spatial correlation. Experimental variograms of factors that are 100 per cent nugget need not modelled and simulated, which can result in a significant time saving.

The MAF transform is available in commercial software (Isatis™).

SUMMARY AND CONCLUSIONS

Linear estimation methodologies such as OK over-smooth the grade tonnage curves when only wide-spaced drilling is available. Non-linear estimation and conditional simulation are alternative approaches that can provide reliable estimates of the recoverable tonnage and grade from wide-spaced drilling. Approaches to assess smoothing using various 'change of support models' have been presented here. Specifically, a well-established non-linear geostatistical approach, the global 'discrete Gaussian model' (DGM) for change of support, is recommended as a tool to establish when to move from linear to non-linear estimates. This approach will materially improve estimate results. In addition, the use of the DGM as a block model validation tool at BHP Billiton's iron operations has been described as a case study. Finally, a synoptic review of non-linear estimation and simulation methodologies applicable to correlated variables has been discussed.

While quantifying smoothing is an important step in deciding when to move from linear to non-linear estimates several additional factors must be considered. Firstly, the end users of the model must be able to deal with the more complicated non-linear estimate or simulation. Secondly, the degree of smoothing must also be weighted against other potential inaccuracies in the model and the stage of project development and thus required accuracy must be weighed up. Finally, the impact of errors due to smoothing must also be compared to the confidence limits of the resource classification.

Isaaks (2004) proposes that in instances other than final grade control estimation (where block estimation accuracy is critical), OK estimates that are deliberately conditionally biased (OKCB)

can provide acceptable local estimation accuracy and accurate estimates of recoverable tonnes and grade above cut-offs. In many instances, local accuracy is not just required for grade control but is also important in medium- to long-term planning. When OK estimates are over-smoothed, non-linear estimation or simulation, not OKCB, is recommended.

Conditional co-simulation, does account for the correlations between variables but is time-consuming to implement. These issues have led to the development of methods such as min/max autocorrelation factors (MAF) and the stepwise conditional transform (SCT). These methods de-correlate variables, thus allowing faster independent conditional simulation.

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